Statics and Dynamics of Disordered Elastic Systems

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We examine here various aspects of the statics and dynamics of disordered elastic systems such as manifolds and periodic systems. Although these objects look very similar and indeed share some underlying physics, periodic systems constitute a class of their own with markedly different properties. We focus on such systems, review the methods allowing to treat them, emphasize the shift of viewpoint compared to the physics of manifolds and discuss their physics in detail. As for the statics, periodicity helps the system to retain a quasi-translational order and to be stable with respect to the proliferation of free topological defects such as dislocations. A disordered periodic system thus leads to a glass phase with Bragg peaks: the Bragg glass. On the other hand, for driven lattices, transverse periodicity allows the system to retain its glassy nature, leading to a moving glass phase. The existence of these two phases has important theoretical and experimental consequences, in particular for vortex physics in superconductors, the physical system which is mainly focused here.

1 Introduction

The statics and dynamics of lines in random media is a long standing problem in the physics of disordered systems. It is one of the remarkable, and experimentally relevant examples of glassy systems with strong analogies but also marked differences with spin glasses. Before the advent of the vortex problem in high Tc superconductors, and the strong motivation to solve it that it entailed, the physics of manifold was viewed as a sort of toy model for more "complicated" and "important" systems such as spin glasses. Indeed, as a purely theoretical problem, disordered models such as the random field XY model, were thought to be at least qualitatively well understood. The discovery of high-Tc superconductors and the new experimental realizations it provided for these systems, shook this belief and led to many new questions. The field was then able to progress at a rapid pace due to the remarkable interplay between the theory and experiments. It has proven to possess a remarkable amount of novel and complex physics, unexpected some ten years ago.

Since the focus of the community was on lines and manifolds, the first theoretical papers on the vortex systems mainly borrowed from this physics. Although this led to spectacular results in the field of superconductors, it also fell short of the mark in some respects, since it misses a good part of the novel physics arising for such *periodic* objects and *entirely* due to the periodic nature of the system. Indeed it was then realized that, both for the statics and the dynamics, periodicity was a crucial ingredient that needed to be treated carefully. Once taken into account, periodicity led not only to new concepts and methods, but also to a radically new physical image, that replaced the previous common beliefs, based on manifold physics. Fortunately, these new theoretical ideas coincided with an experimental maturation of the field: experiments became accurate, the main spurious effects were fairly

well understood, so that experimental results are now firm and unavoidable. They put stringent constraints on whatever theoretical interpretation can be proposed, well above the level that can be satisfied by vague and very often self-contradictory "handwaving" arguments, and provide a very strong stimulus for accurate (and correct) theories.

Since a number of reviews already exist in this field, whether for the statistical mechanics of the manifold point of view [1], or centered on the physics of vortex systems [2,3], this paper specially develops the new results, concepts and methods, derived in the last four years, following from the treatment of periodicity. Since the main actor in this progress are the vortex systems, we first start in section 2 by a very basic introduction to the physics and important questions for such vortex lattices. We also briefly review the various theoretical approaches put forward to tackle this problem. We then focus in section 3 on the general problem of the statics of a periodic structure. Although such a study applies to the vortex lattice it has many other physical applications that we also briefly discuss. A section is devoted to the interesting case of $d = 2$. section 4 is devoted to the dynamical effects. Here again periodicity plays a crucial role. After having reviewed the basic methods and results for the dynamics of simple manifolds we concentrate on these novel effects. Finally some conclusions and open questions can be found in section 5.

2 Vortex physics

2.1 Experimental questions

The conventional mean field phase diagram [4] of type II superconductor consists of a Meissner phase ($H < H_{c1}(T)$) and a mixed phase ($H_{c1}(T) < H < H_{c2}(T)$). In the mixed phase the magnetic induction B can penetrate the bulk of the superconductor in the form of vortex lines each carrying a quantum of magnetic flux $\Phi_0 = hc/2e$ and aligned along the external field H. A vortex consists of a normal region of radius ξ_0 where the superconducting order parameter Ψ vanishes, surrounded by a region of size λ where supercurrents screen the external field. By minimization of the Landau-Ginzburg functional, Abrikosov predicted [5] that these vortex lines form a regular lattice (triangular in standard systems), later observed. This lattice can also be described as a standard lattice with elastic coefficients [6,2]. From flux quantization, the lattice spacing is simply related to the field $a \sim \sqrt{\Phi_0/B}$. In high T_c materials it can vary over several orders of magnitudes typically from $a \sim \lambda \sim 0.5\mu m$ near H_{c1} to $a \sim \xi_0 \sim 10A$ near H_{c2} (outside the critical regions).

Even for a pure system (i.e. without disorder), real life is more complicated and, as can be expected from a lattice the Abrikosov vortex lattice melts on a line $H_m(T)$ below $H_{c2}(T)$, as predicted a long time ago [7]. However it is only with the advent of high-Tc superconductors, where $H_m(T)$ is expected to lie well below $H_{c2}(T)$ due to higher anisotropy and temperatures, that such effects were studied in details [2,8,9]. The high temperature phase is a vortex liquid, which in some regimes can be thought of as a collection of fluctuating entangled lines [8,10,11]. For the pure system, it is now reasonably established, mainly through numerical simulations [12,13] that this transition occurs and is first order. The detailed theory of this transition is difficult and still controversial since it should describe both the formation of the Abrikosov lattice and the fluctuations of the superconducting order parameter [14,15].

An additional but necessary complication when dealing with any superconducting phase is quenched disorder, as can be seen from dynamical considerations. Indeed, in the presence of a current J, each vortex line segment dl is submitted to a Lorentz force $F = (\Phi_0/c)J \wedge dl$. In the absence of any source of pinning the vortices will start moving in the direction perpendicular to the current. This motion will in turn generate an e.m.f. (a voltage drop) along the same direction than the current and lead to dissipation: the material will then be a rather poor conductor whose resistivity will be $\rho_{\rm ff} = \rho_N(B/H_{c2})$ (the so called flux flow resistivity) where ρ_N is the normal state resistivity. Disorder, on the other hand provides preferential regions for the vortex cores to sit in and thus prevents the vortex lattice from sliding freely and dissipating. Thus, somewhat paradoxically, some amount of quenched disorder is crucial to make the material a superconductor (it is a fine balance since too much disorder is detrimental again by destroying superconductivity altogether). At $T = 0$, the pinning by disorder leads to the existence of a critical force F_c (corresponding to a critical current density J_c), below which vortices stay still, the average velocity is zero and no dissipation occurs. Disorder already exists in any real material either in the form of microscopic defects such as oxygen vacancies (*uncorrelated disorder*) or more macroscopic defects such as twin planes (*correlated disorder*). Since it is obviously very important for technological purposes to increase the critical current and optimize pinning, various methods have been studied to increase disorder, such as electron irradiation, which creates more uncorrelated point disorder, or heavy ion irradiations, which creates parallel columns of defects (columnar disorder). Columnar defects have been found to be particularly efficient to pin the vortex lines [16,17] and their effect can be shown to be further optimized by deliberately crossing them in splayed configurations [18,19]. The physics of correlated disorder and its connections to quantum disordered problems has also brought about many new and interesting developments but goes beyond this paper (see e.g Refs [20,21,2]).

Again, before the current interest in high Tc materials, the dynamics of superconductors was treated in an oversimplified way. Some of the main experiments to be explained were (i) transport experiments, i.e the shape of the I-V characteristics and (ii) magnetization relaxation experiments, i.e the relaxation of the magnetic field profile inside a sample due to thermally activated flux motion (flux creep). The motion far above the threshold was supposed to be simple since the vortices slide very fast, and average very well over the disorder. One expects to find $v = F/\eta$, where η is the friction coefficient, v is the average velocity (and thus voltage) and F the applied force (proportional to the current J). Motion around and below the threshold was described by the conventional Anderson-Kim model, which was sufficient to account for most of the observations [4]. This was an effective one particle model, where an unspecified piece of the vortex lattice (a vortex bundle), moves as a single particle in a one dimensional potential energy landscape (tilted by the applied Lorentz force). The potential could be chosen with some amount of disorder or periodic because of the periodicity of the lattice. At $T = 0$ this model immediately yields a critical force F_c corresponding to the maximal slope in the landscape. At $T > 0$ thermal activation allows motion by overcoming the barrier U_b needed to go from one minimum to the other. This simple model yields:

$$v \sim \rho_{\rm ff} F e^{-U_b/T} \tag{1}$$

At any finite temperature one would thus recover an exponentially small but *finite* resistivity (i.e vortex mobility) $\rho \sim \exp[-U_b/T]$ (we recall that the current density is $J \sim F$, the voltage drop $V \sim v$). This mechanism, known as Thermally Assisted Flux Flow (TAFF) thus focuses on the motion of individual vortices (or vortex bundles seen as a single point). Thus in this conventional approach the mixed phase is not a true superconductor at any finite temperature, since it possesses a finite (albeit exponentially small) linear resistivity. Also this model was insufficient to obtain any estimate for J_c, which had to wait for more sophisticated approaches by Larkin and Ovchinikov [22]. The simple TAFF approach became totally insufficient in high T_c materials where giant thermal flux creep effects were soon observed. It became apparent that at low temperature the above TAFF law should be replaced by

$$v \sim \rho_{\rm ff} F e^{-U_b(J)/T} \tag{2}$$

with an effective barrier $U_b(J)$ exhibiting a strong dependence on the current, increasing rapidly with decreasing J. Such a current dependent barrier could not be explained without a more sophisticated theory taking into account the elasticity and periodicity of the flux lattice.

A similar situation also occurred for the static properties of the vortex lattice. Paradoxically, although Larkin soon realized that even weak disorder would have a strong impact, and lead to a destruction of the perfect translational order [23], such effects were not investigated in detail until very recently. In fact only after the discovery of high Tc superconductors was it realized that disorder plays a crucial role. Experimentally, it was observed very early that the phase diagram is very different from the predictions based on the ideal case (with only a melting transition between a perfect solid and a flux liquid). Instead there is an *irreversibility line* [24] $H_{irr}(T)$ below which the system seems to exhibit a glassy behavior, as can be seen from the history dependence of physical quantities, non linear I-V with vanishingly small linear resistivity and irreversibility. To understand the physical properties of what was the solid phase, both from the point of view of statics and dynamics, it is thus necessary to take disorder into account. Disorder will also affect the nature of the transition to the vortex liquid phase. The first experiments [25] concluded that the irreversibility line corresponded to a *continuous transition*. However, it became increasingly clear in later and more precise experiments starting with observations of jumps in the resistivity [26], that the transition at low fields and weak disorder is in fact a *first order transition*. The first order nature of the transition at low fields is by now well established by a variety of techniques, such as transport measurements [26,27], magnetization jump [28] or specific heat measurements [29].

Thus in view of the new experimental results, the limitations and inadequations of the conventional theories became clear. Both for the technological applications of high-T_c materials and from a purely theoretical point of view, it was thus of paramount importance to understand the detailed properties of the disordered vortex lattice.

2.2 Different theoretical approaches

Both experiments and analogies with other disordered systems in statistical mechanics known to exhibit glassy effects, suggest that the disordered vortex system too could lead to the formation of a glassy state rather than that of a vortex lattice. Although the nature of such a state was unclear, one of its key properties should be, as in any glassy system, to possess many low lying metastable states, with barriers between them which diverge as a function of their "separation" in phase space. Thus the low temperature phase, if it really is a "true glass", should be characterized by the true vanishing of the linear resistivity even at finite temperature [30,31]. Or if it is only a glass in an approximate way (with only finite barriers) it should still lead to extremely small linear resistivity. This is thus a significant departure from the above models of thermally assisted flux flow, which assumed *finite* barriers between pinned states. A sign of an instability upon increasing disorder, presumably towards a glass, was also found in the flux liquid [11].

Divergences between different theoretical approaches appeared in the way of describing this glass phase. Of course the full description is given by a disordered Ginzburg-Landau functional. However such a theory is too complicated to be analytically tractable. One way to proceed then is to assume, as a phenomenological description, that a complete destruction of the Abrikosov lattice occurs even at very short length scales [30,32]. Such an approach was prompted by an attempt to interpret the existing decoration pictures at that time, and early experiments showing a continuous phase transition. It was then proposed [30], originally by analogy with the Cardy Ostlund 2D disordered XY model [33] (a rather stretched analogy as it turns out - see Section 3.5), that there should be a low temperature phase, called the "vortex glass" with true zero linear resistivity. To put some flesh on this rather handwaving derivation of the physical properties, it was later proposed [32] that the disordered Ginzburg-Landau model could be approximated (while keeping the main ingredient of Ref. [30], namely the absence of a lattice) by a simpler discrete XY model (in the superconducting order parameter) with a quenched random gauge field. As shown by numerical simulations this "gauge glass model" leads to a zero linear resistivity in $d = 3$, a continuous "vortex glass" transition (with scaling of the I-V curve near T_g). The physical picture was thus of an effective "spin glass" order in the superconducting order parameter.

The second approach is completely different. It retains the elastic lattice structure at small scale [31] and describes vortex lines as strings having some elastic energy. Disorder is then incorporated as acting directly on these elastic classical objects, which amounts to forgetting about the phase of the order parameter beyond the elastic theory. The vortex problem thus becomes a particular case of the more general problem of an elastic system in the presence of disorder.

Although different in nature, both theories agreed that the disorder essential to produce the glassy low temperature phase and the vanishing of the linear resistivity, was also destroying at large scales the perfect flux lattice existing in mean field theory. The low temperature phase was therefore generally expected to be a topologically disordered phase, lacking translational order. Several calculations supported this point of view. Elastic theory predicted at best a stretched exponential decay of translational order [31,34,35]. In addition general arguments tended to prove that disorder would always favor the presence of dis-

locations [32,36]. The vortex lattice seemed to be buried for good. However, although these approaches seemed to explain some aspects of the problem, various others did not naturally fit in the framework of these theories. As already mentioned, experimentally the transition between the glass phase and the liquid is first order at low fields [26,27] rather than continuous transition observed at high fields which is predicted by the gauge glass model. Furthermore, decoration experiments of the flux lattice at very low fields (60 G) in several materials were showing remarkably large regions free of dislocations [37] inconsistent with the assumptions behind the "vortex glass". Efforts to improve on the gauge glass model by incorporating screening effects showed in numerical simulations [38] that the "vortex glass" phase would *not exist* in $d = 3$ (the lower critical dimension $d_{lc} > 3$). There was also disagreement within a purely elastic description: old calculations on the related disordered elastic random field XY model [36] as well as more recent scaling arguments for the vortex lattice [39] suggested, within a purely elastic description, a slower, logarithmic growth of deformations. All these problems, both theoretical and experimental, prompted for the need of a quantitative theory of a disordered vortex lattice. Before we look at it, let us further examine the consequences of an elastic description of the vortex lattice.

3 Statics of lattices with disorder

Let us now follow the route of the elastic theory. Such an approach has the advantage over the gauge glass approach to at least allow for some analytic calculation, and is certainly a good starting point if the disorder is weak. Of course the stability of the elastic approximation to topological defects has to be (and will be) examined in the end. Besides applying to vortex lattices, such disordered elastic systems also cover many physical situations, such as charge density waves [40], Wigner crystals [41] magnetic bubbles [42,43], Josephson junctions [44,45], the surface of crystals with quenched bulk or substrate disorder [46] and domain walls in incommensurate solids [47]. All these systems have in common a perfectly ordered underlying structure modified by elastic distortions and possibly by topological defects such as dislocations, due to temperature or disorder. As for the vortex lattice, for many of such systems the periodic structure can be set in motion by an external force (e.g. and electric field for the CDW or the Wigner crystal), and the velocity can be measured (e.g. by measuring the current for CDW or Wigner crystal).

3.1 Description in term of an elastic theory

Let us now look at the minimal model describing these different physical systems. First, the pure problem: we can ignore the internal structure of the objects (vortices, magnetic bubbles etc.) and represent them as point-like objects. One can then define an equilibrium position R. At that point one has to distinguish between manifolds of internal dimension d and periodic structures. For the manifold one defines displacements u (vectors of dimension N) relative to the equilibrium position. A manifold is thus naturally embedded in a space of dimension $D = N + d$ (for instance a $d = 2$ interface in a $D = 3$ space). It has an elastic

energy

$$H_{\rm el} = \frac{1}{2}\sum_{\alpha,\beta}\int \frac{d^d q}{(2\pi)^d} u_\alpha(q)\Phi_{\alpha\beta}(q)u_\beta(-q) \tag{3}$$

The $\Phi_{\alpha\beta}(q)$ is the elastic matrix. We can also rewrite (3) symbolically in real space as

$$H_{\rm el} \sim \frac{1}{2}\int d^d r \;\; c\,(\nabla u)^2 \tag{4}$$

where c is the elastic coefficient (here isotropic for notational simplicity). For periodic systems the situation is more subtle. The equilibrium positions are discrete R_i (at least with respect of some of the coordinates), and the lattice spacing is a genuine scale of the problem. The elastic energy should depend on the discrete differences between the displacements of two objects, such as $u_i - u_{i+1}$. If the relative displacements of two neighbors is small, a situation realized at low temperature and small disorder, one then usually takes the continuum limit by letting the lattice spacing a go to zero, or one performs a quadratic expansion in u to obtain an elastic energy similar to (3), but where the sum over q is restricted to the first Brillouin zone. In this continuous limit manifolds and periodic systems would thus superficially look very similar from the point of view of their elastic energy. However, this continuum limit should be performed with great care since disorder can also vary at scales *much smaller* than the lattice spacing a. Therefore the existence of typical scale a in the periodic structure must have some impact on the physical properties of the system. As we will see it is hidden in the expression of the density of objects as a function of the displacements. Note that after averaging over disorder only *relative displacements* have a direct physical meaning since disorder is statistically translationally invariant and one can obviously translate the whole lattice without changing the elastic energy.

Two important quantities characterize the physics of such an elastic system. The first one measures the relative displacements of two points (e.g two vortices) separated by a distance x.

$$\tilde{B}(x) = \frac{1}{N}\overline{\langle [u(x) - u(0)]^2 \rangle} \tag{5}$$

where $\langle\rangle$ denotes an average over thermal fluctuations and $\overline{}$ is an average over disorder, and N is the number of components of u. The growth of $\tilde{B}(x)$ with distance is a measure of how fast the lattice is distorted. For thermal fluctuations alone in $d > 2$, $\tilde{B}(x)$ saturates at finite values, indicating that the lattice is preserved. Intuitively it is obvious that in the presence of disorder $\tilde{B}(x)$, will grow faster and can become unbounded. $\tilde{B}(x)$ can directly be extracted from a direct imaging of the lattice, such as performed in decoration experiments.

Related to $\tilde{B}(x)$, albeit different, is the structure factor of the lattice, obtained by computing the Fourier transform of the density of objects:

$$\rho(x) = \sum_i \delta(x - R_i - u_i) \tag{6}$$

The square of the modulus $|\rho_k|^2$ of the Fourier transform of (6) is measured directly in diffraction (Neutrons, X-rays) experiments. For a perfect lattice the diffraction pattern consists of δ-function Bragg peaks at the reciprocal vectors of the lattice. If some degree

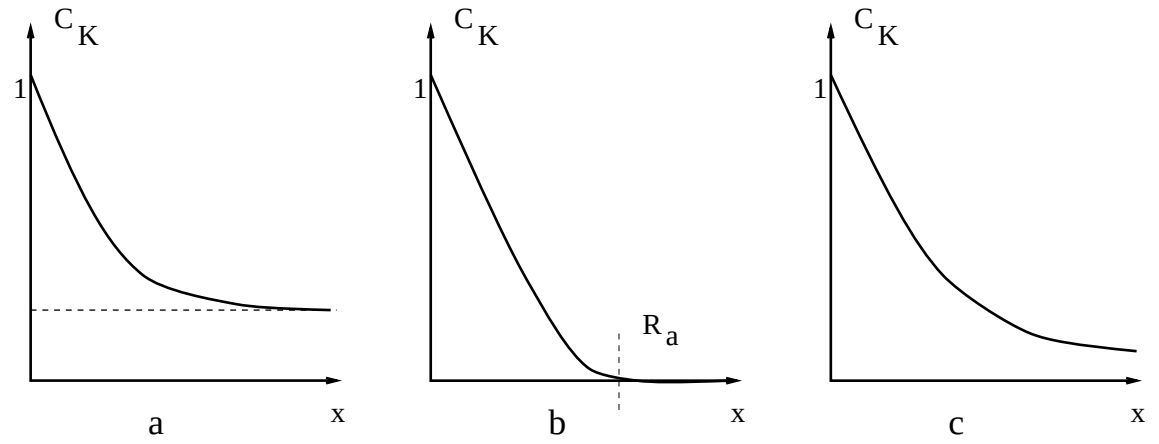

Figure 1: Various possible decays of $C_K(x)$. (a) For thermal fluctuations alone $C_K(x) \to$ Cste, one keeps perfect δ function Bragg peaks, albeit with a reduced weight (the Debye-Waller factor). (b) $C_K(x)$ decays exponentially fast. The structure factor has no divergent peak any more, so translational order is destroyed beyond length R_a, although some degree of order persists at short distance. (c) $C_K(x)$ decays as a power law. The structure factor still has divergent peaks but not sharp δ function ones. One retains quasi-long range translational order. This is for example the case in $d = 2$ at small temperature (Kosterlitz-Thouless).

of short range order persists, individual peaks will still be present although they might be broadened, and will not be simple δ-functions any more. The shape and width of any single peak is thus again a measure of the degree of translational order in the lattice. To be more quantitative, one can filter a single peak centered around the reciprocal vector K and Fourier transform it back, to obtain a correlation function in real space. This correlation function (called the translational order correlation function) is given by

$$C_K(x) = \overline{\langle e^{iKu(x)} e^{-iKu(0)} \rangle} \tag{7}$$

Clearly the broader the peak the faster the decay of $C_K(x)$. $C_K(x)$ is therefore a direct measure of the degree of translational order that remains in the system. Three possible cases are shown in figure 1. For simple Gaussian fluctuations

$$C_K(x) = e^{-\frac{K^2}{2}\tilde{B}(x)} \tag{8}$$

but such a relation holds only qualitatively in general.

Of course one should also go beyond the simple elastic approximation (3) and worry about the possible existence of topological defects such as dislocations. We will come back to this point later. If unpaired dislocations are present they will destroy the translational order exponentially fast beyond a certain length scale R_D of the order of the typical distance between such unpaired dislocations.

Although coupling to disorder depends on the precise microscopic aspects of each system, it is usually possible to model it by pins put at random positions in the system and coupling to the density.

$$H_{\text{rand}} = \int d^d x V(x) \rho(x) \tag{9}$$

The disorder potential is $V(x) = U_0 \sum_i \delta(x - x_i)$ where x_i are the positions of the defects. The δ-function should be understood here as a short range correlation function of range r_f. In superconductors one has roughly $r_f \sim \xi_0$ which can be much smaller than the lattice spacing a. The effect of disorder depends on the relative strength of the pins and the elastic forces. If the pins are very strong, one must retain their discrete nature. On the other hand if disorder is weak (for the regime of validity of such an approximation see [21]: in $d \geq 2$ such an approximation always holds provided the strength of each pin is weak), pinning occurs at lengths much larger than the average defects distance (this notion will be made more quantitative in section 3.2). It is then legitimate to replace V by a simple Gaussian potential with a correlator

$$\overline{V(x)V(x')} = \Delta(x - x') \tag{10}$$

where Δ is a function of range r_f. (3,9) describes the most general elastic system coupled to disorder. Even with the Gaussian disorder V (3,9) is a rather formidable theory to solve because of the highly non-linear nature of the coupling to disorder.

Before attacking the problem with the full force of replica theory and renormalization group, let us examine some simple arguments to understand its physics. This is instructive, since a good part of the physics can be derived simply, but also shows how too simple pictures can also prove misleading when used beyond their range of validity.

3.2 Larkin model

Since even Gaussian disorder is too complicated to be studied directly, Larkin had the remarkable idea [23] to replace the coupling to the random potential (9) by random forces acting on the vortices and coupling directly to the displacements

$$H_{\mathrm{L}} = \int dx f(x) u(x) \tag{11}$$

where f has Gaussian correlations. Being linear (3-11) is now rather straightforward to solve. It yields a displacement correlation function of the form

$$\tilde{B}(x) = l^2 (x/R_l)^{4-d} \tag{12}$$

where l is any lengthscale and R_l the distance for which the relative displacements are of order l. Since the theory is simply Gaussian, $\tilde{B}$ and C_K are simply related by (8). Disorder thus destroys the long range translational order below four dimensions, and leads to an exponential decay of the correlation function in the physically relevant $d = 3$ case. It is important to note that the Larkin model has *two* characteristic length scales:

- R_c which is the distance in the XY-plane at which the relative displacements of the flux lattice are of order the *correlation length* of the random potential, r_f (which is of order ξ_0 at low temperature), and
- R_a, which is larger, and is the distance over which the relative displacements of the flux lattice are of order the *lattice spacing* a.

R_a is related to the disorder strength by (17) below, and R_c is given by a similar expression but with a replaced by ξ_0. Using (8), R_a can be related to the length beyond which translational order is really destroyed. The naive physical picture emerging from this model is the one of a crystal breaking into crystallites of typical size R_a, as schematized in case (b) of figure 2, and would correspond for the translational correlation function to case (b) of figure 1.

However this model has several limitations built in the approximation (11) of the random term. Since it lacks the translation symmetry $u \to u + a$ of the physical model (9), it gives a real meaning to the displacement field u itself, whereas the original model can of course be sensitive only to the *position* of the line itself. The Larkin model can thus be viewed as an expansion in powers of the displacements u, and only holds when the displacements are small. That the approximation (11) is too crude is also apparent when one tries to determine the pinning force. Since the Larkin Hamiltonian is *linear* in u, any global translation of u does not change the average energy, and thus the pinning force is zero. In order to describe correctly the pinning one *needs* the nonlinearities of (9).

In a masterful stroke of physical intuition, Larkin and Ovchinikov (LO) realized [22] that the breakdown of validity of the Larkin model occurred exactly at the lengthscale corresponding to the critical pinning force. Indeed one can associate to an applied force F, a lengthscale in the static problem corresponding to the size of the smallest bundle moving. This size can be obtained by balancing the elastic plus pinning energy with the Lorentz force work. Since the pinning energy grows only as $L^{d/2}$ and not as the volume, a small external force will be able to move only large bundles (we will come back to this point in section 3.3), thus the smaller the force, the larger the size of the bundle. As long as the Larkin model is valid, no pinning exists and the lattice flows "freely" and one should be *above* F_c. It is only below F_c, i.e. for length scales larger than those for which the Larkin model applies that one can expect anomalous transport. One may think naively that the expansion breaks down at R_a. In fact this occurs much sooner at R_c, as was realized by LO: the expansion in displacements becomes incorrect as soon as a line can be considered off its equilibrium position, i.e. when it has moved by more than its intrinsic width roughly given by the core size ξ_0. More precisely the expansion breaks down at scales for which the allowed relative motion is such that the random potential cannot be approximated by its slope only. The critical force is thus associated to the typical energy for which motion of the lines is of order $r_f \sim \xi_0$. If $R_c \gg a$, this energy is obtained by balancing the energy gained due to the applied force $\sim u R_c^d F_c$ to the typical pinning energy $E_{\rm pin}$ of a bundle of size R_c. Such a bundle has an elastic energy $E_{\rm el} \sim c R_c^{d-2} u^2$ and a pinning energy $E_{\rm pin} \sim R_c^{d/2} \Delta^{1/2} u$ of the same order $E_{\rm el} \sim E_{\rm pin}$ for displacements $u \sim r_f$. The critical force density is thus given by

$$F_c = \frac{c r_f}{R_c^2} \tag{13}$$

Formula (13) is quite remarkable since it makes it possible to obtain one of the most interesting physical quantities for the dynamics directly out of length scales of the *static* problem. It also shows that that simple TAFF arguments are too simple: in order to correctly describe the effect of the external force and hence the motion, the *collective* behavior of the lines

should be investigated. Indeed the motion of individual lines would lead to a cost in energy much too high compared to the energy gained due to the motion. On the other hand, the motion of large enough bundles are able to overcome the pinning force. Determination of the transport below F_c thus requires the knowledge of the static properties of the lattice at lengths larger than R_c, for which a solution of the nonlinear problem (9) is required.

3.3 Characteristic lengthscales

In order to gain a physical insight in the static properties of the disordered lattice one can use simple dimensional analysis. In the presence of many weak pins, u cannot adapt to take advantage of each of them, due to the cost in elastic energy. One can assume that u varies of $\sim a$ over a length $R_a \gg a$. The density of kinetic energy is $\sim c(a/R_a)^2$, where c is an elastic constant. For the disorder (9), one has to be very careful to separate its various Fourier components. Indeed the period of lattice introduces a *natural* scale a. The Fourier components of the disorder close to vector of the reciprocal lattice thus play a special role, as appears when one rewrites the density (such a decomposition is *exact* in the elastic limit[48,49])

$$\rho(x) \simeq \rho_0(1 - \partial_\alpha u_\alpha(x) + \sum_{K \neq 0} e^{iKx} e^{-iK\cdot u(x)}) \tag{14}$$

The gradient term is simply the standard change of density due to a compression of dilatation, whereas the other terms correspond to a translation of the lattice without any change in the average density. This decomposition of the density is very similar to the density modulation in a CDW, where the role of u is played by the phase of the CDW, and only one K vector exists.

If one assumes that u is roughly constant over a length R_a, the Fourier components of the disorder with components close to e^{iKx} will give different contributions. It is easy to see that the long wavelength part of the disorder can only be relevant for $d \leq 2$, and thus can be safely dropped. The main contribution comes from the Fourier modes of the disorder which have a periodicity close to the one of the lattice, and which do not tend to change the local density but rather to shift the lattice locally:

$$H^{\rm dis}_{q\sim K} = \rho_0 \int d^d x V(x) e^{iKx} e^{-iKu(x)} \tag{15}$$

This sum can be viewed as a random walk in the complex plane[50] and the value of u adjusts itself to match the phase of the random potential. Therefore the gain in energy density due to the disorder term is of order

$$H^{\rm dis}_{q\sim K} \sim \Delta_K^{1/2} / R_a^{d/2} \tag{16}$$

where Δ_K are the Fourier modes of the disorder correlator (10), for vectors of the reciprocal lattice. Minimization gives a length R_a of the order of

$$R_a \sim a \left(c^2 a^d / \Delta_K \right)^{1/(4-d)} \tag{17}$$

Weak disorder is thus relevant below $d = 4$. Beyond the length R_a the relative displacements are larger than the lattice spacing a. One can thus naively see the lattice as broken into domains of size R_a above which the translational order would be destroyed (displacements are larger than a). This image would be in agreement with the one one gets from the Larkin model (used *beyond* its range of validity), and as such has been often accepted as correct. As we will see this vision is too naive, but the length R_a is indeed one important lengthscale of the problem. Way below R_a, one could also identify R_c with the length for which the displacements are of order ξ_0. The same analysis, but with $u \sim \xi_0$ would give $R_c \sim \xi_0 \left(c^2 \xi_0^d / \Delta_K\right)^{1/(4-d)}$. It is important to note that the above arguments give only the way these two lengths depend on disorder. Obviously $R_c < R_a$ and in fact one can see that $R_c/R_a \sim (a/\xi_0)^{1/\nu}$, where ν is the roughening exponent defined by $B(x) \sim x^{2\nu}$, or equivalently by saying that the displacement u scale as $u \sim L^\nu$. For the Larkin model (12) implies $2\nu = 4 - d$. Two kinds of domains can thus be defined. For length smaller than R_c the Larkin model is valid and the displacements grow as $x^{(4-d)/2}$ giving (12). Although this behavior can be extracted from an exact solution of the Larkin Hamiltonian it can also be obtained by simple dimensional analysis. If one assumes that $u \sim L^\nu$, balancing the elastic and disorder terms in the Larkin Hamiltonian gives

$$L^{d-2+2\nu} \sim L^{d/2+\nu} \tag{18}$$

thus yielding $u \sim L^{(4-d)/2}$. Note that R_c is in fact the *only* length that can be extracted from the Larkin model and that gives the bundle size corresponding to F_c. It is important to carefully distinguish between R_c and R_a, since they correspond to two physically different lengthscales. Serious confusion exists in the literature in this respect (see e.g. [51,52]).

What happens beyond the LO length R_c is more subtle and has only been understood recently. Since the lines have moved by more than their intrinsic width, one cannot use Larkin's random force approximation, and the full nonlinear Hamiltonian (9) should be used. Intuitively one can understand this regime provided that the displacements remain small compared to the lattice spacing, i.e. for separations smaller than R_a. Indeed in that case each line sees it *own* realization of the random potential. One is thus led back to the random manifold problem, where each point of the manifold sees an independent random potential, since it corresponds to different internal manifold coordinates. To obtain a simple estimate for the growth of displacements one can again balance the elastic and random energy, but this time keeping the potential term (9). Similar scaling arguments give a mean-field (Flory) roughening exponent $u \sim L^{\nu_F}$ with $\nu_F = (4-d)/(4+N)$ (d being the space dimension and N the number of components of the displacement field as defined in section 3.1). This exponent is not exact but a good approximation of the true random manifold exponent $\nu_{\rm rm}$. Contrary to what happens in the Larkin regime, the random manifold regime corresponds to a glassy regime, where the system can find many metastable states.

A manifestation of this glassy regime can be found in transport, since as already mentioned the detailed transport properties depend on how the lattice is pinned. To estimate the $v - F$ characteristics one can use the near equilibrium collective creep arguments [31], balancing the energy gained due to the external force with the energy of the pinned system. If the system is in the ground state the closest metastable state of a bundle of size L scales

as L^θ (where θ is the energy exponent $\theta = d - 2 + 2\nu$). The standard assumption is that energy barriers scale with the same exponent $\psi = \theta$ (it was possible in some cases to check it explicitly). In order to reach this metastable state the energy loss should equal the gain due to the external force giving

$$L^\theta \sim L^d L^\nu F \tag{19}$$

The smallest bundle to move will have a size $(1/F)^{1/(d+\nu-\theta)}$ and an energy $E = (1/F)^{\theta/(d+\nu-\theta)}$. The velocity will be non-linear and given by

$$v \sim e^{-\frac{U_c}{T}(F_c/F)^{\theta/(d+\nu-\theta)}} \tag{20}$$

where U_c is the barrier at scale R_c. The existence of many metastable states separated by diverging barriers thus manifests itself in the vanishing of the *linear* resistivity. This is different from the naive TAFF picture (which corresponds to finite barriers). Thus one of the main achievements of the collective creep picture is to account for the existence of true superconductivity. For that the needed ingredients were elasticity and disorder. It is interesting to note that the Anderson-Kim model can be generalized [53] (and solved exactly) to an arbitrary one dimensional landscape. One can then recover a lot of the features of these glass phases, and of the vortex glass transition by choosing a landscape with *long range correlations* which captures the diverging barriers in a phenomenological way.

However even below R_a the validity of this elastic approach was questioned. Indeed the entire above study completely ignores the possibility of creating topological defects, either in the statics or for the dynamics. These defects could ruin the above nice result (20) and the very existence of a true glass phase. In the absence of disorder and at low temperature it is of course obvious that defects able to destroy the translational order, such as unbounded dislocations, cannot appear. In the presence of disorder, the issue was less clear. A long line of arguments, going back to [36] and further developed in [32] were put forward to indicate that in the presence of disorder, however weak, dislocations should always proliferate. Since the elastic theory already "showed" that long range order should be destroyed beyond R_a these arguments were not challenged in subsequent studies which addressed the question of the behavior of the elastic deformations [39,2]

As we will show such arguments were incorrect, though worth examining. An Imry Ma type argument is the following. The core energy cost of a dislocation cannot be avoided and scales as L^{d-2}. A dislocation loop of size L creates extra-displacements of order O(1) up to logarithms, in a region of size L^d. By adjusting the position of the loop one can hope to gain an energy from disorder $L^{d/2}$. Thus below $d = 4$ large (infinite) dislocation loops will be favorable. The argument is flawed because it is again implicitly assuming the same physics as in Larkin's random force model for which the disorder energy is linear in the displacement. For the real model (15) the energy varies as $\cos(Ku)$; adding a dislocation displacement will not necessarily gain enough disorder energy. We will come back to this point and give the correct arguments in section 3.4. The issue of the existence of dislocations is an important one. Indeed if dislocations are generated by disorder, the elastic theory does not hold and its conclusions are questionable. In particular, if dislocations are present the non-linear $v - F$ characteristic should be replaced by a TAFF characteristic due to the plastic deformations.

3.4 Full solution of the periodic problem

Let us now examine the full solution of the problem, and in particular examine the physical properties beyond the length R_a for which the *periodicity* of the lattice becomes important. To do so we use a variational formulation of the problem [48,49,54]. Similar results have been obtained using the functional renormalization group approach [48,49].

A variational formulation

In order to average over the disorder in (3,9), we use the replica trick and obtain

$$H_{\rm eff} = \frac{c}{2}\int d^d x (\nabla u(x))^2 - \int d^d x \sum_{a,b} \sum_{K\neq 0} \frac{\rho_0^2 \Delta_K}{2T} \cos(K\cdot(u^a(x) - u^b(x)))] \qquad (21)$$

of course the full elastic Hamiltonian should (and has) been used, the above being a simplified notation. In particular for vortex lattices the anisotopy introduced by the magnetic field between the in-plane and along the field directions can be trivially treated by a rescaling (for more details see [49]). We now look for the best trial Gaussian Hamiltonian H_0 in replica space which approximates (21). It has the general form [55]

$$H_0 = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d} G^{-1}_{ab}(q) u_a(q)\cdot u_b(-q) \qquad (22)$$

where the $[G^{-1}]_{ab}(q)$ is a n by n matrix of variational parameters. We obtain by minimization of the variational free energy $F_{\rm var} = F_0 + \langle H_{eff} - H_0\rangle_{H_0}$ the saddle point equations for the variables G^{-1}. The technical details of the solution can be found in [48,49]. Such an approximation is expected to be a good one unless kink excitations around the pinned configurations are very important. As can be confirmed by an independent functional renormalization group calculation [48,49] (in $d = 4-\epsilon$ dimensions), the variational approach seems to capture here the correct physics.

Two general classes of solutions can exist for (22). One preserves the symmetry of permutations of replica, and amounts to mimic the distribution (thermal and over disorder) of each displacement mode $u(q)$ by a simple Gaussian. The other class, which is a better approximation in the glass phase, breaks replica symmetry and approximates effectively the distribution of displacements by a hierarchical superposition of Gaussians centered at different randomly located points in space according to a Parisi scheme which is described in detail in [55]. As can be expected the most stable solution is the one that breaks replica symmetry (a full RSB for $2 < d < 4$), confirming the glassy properties. On the variational solution the two lengthscales R_c and R_a also appear and define three regimes as a function of the separation r which will be discussed below. In the present problem the physics contained in the RSB solution can be expressed as follows: each Gaussian at the lowest level of the hierarchy is associated to a different metastable "pinned" position of the manifold, corresponding to the Larkin length R_c. Let us illustrate it, for simplicity, on the simplest case of a one step RSB solution, which is the solution in $d = 2$ (we thus anticipate on section 3.5). The double distribution over environment and thermal fluctuations is approximated as

follows. In each environment there are effective "pinning centers" corresponding to the low lying metastable states (prefered configurations). Since all q modes are in effect decoupled within this approximation, for each q mode a prefered configuration (a state) is $u_\alpha(q)$. They are distributed according to:

$$P(u_\alpha(q)) \sim \prod_\alpha e^{-\frac{c}{2T_g}q^2|u_\alpha(q)|^2} \tag{23}$$

Each is endowed with a free energy f_α distributed according to an exponential distribution $P(f) \sim \exp(u_c f/T)$ (here $u_c = T/T_g$). Once these seed states are constructed, the full thermal distribution of the q mode u_q is obtained by letting it fluctuate thermally around one of the states:

$$P(u_q) \sim \sum_\alpha W_\alpha e^{-\frac{c}{2T}(q^2+R_c^{-2})|u_q-u_\alpha(q)|^2} \tag{24}$$

where each state is weighted with probability $W_\alpha = e^{-f_\alpha/T}/\sum_\beta e^{-f_\beta/T}$.

One thus recovers qualitatively the picture of Larkin Ovchinikov as the solution of the problem with the replica variational method. The LO length naturally appears as setting the (internal) size of the elastically correlated domains. The full RSB case corresponds to more level in this hierarchy of Larkin domains (in some sense there are clusters of domains of size larger than R_c) and the way this hierarchy scales with distance reproduces the Flory exponents for displacements and energy fluctuations.

The other important method which provides a picture consistent with this one is the functional renormalization group FRG developed by D.S. Fisher[56,57]. There it is found that beyond R_c a non-analyticity develops in the coarse grained renormalized disorder correlator. This corresponds to the renormalized random potential developing cusp singularities [58] (consistent with the LO picture of the medium breaking into Larkin domains). The FRG has the advantage to take better non linearities into account, but it does work only near $d = 4$. The variational method on the other hand works in any dimension as an approximation. Since it is a Hartree replica method it does becomes exact when $N \to \infty$ for the manifold problem. In the large N solution the various pure states u_α do not talk to each other (there is true breaking of ergodicity). Presumably this should be improved by instanton type contributions for realistic finite N, though how to do this remains a totaly open question. Even if taken simply as an approximation and with a grain of salt the GVM gives however a very reasonable physical picture.

Full solution: the three regimes

For point-like disorder, there are *three* different regimes. The variational approach predicts the full crossover function between three regimes. There are as follows:

i) When $\tilde{B}(r)$ is shorter than the square of the Lindemann length $l_T^2 = \langle u^2 \rangle$, the thermal wandering of the lines averages enough over the random potential and the model becomes equivalent to Larkin's model for which $\tilde{B}(r) \sim |r|^{4-d}$. At low temperature, l_T is replaced by the superconducting coherence length ξ_0 (i.e. the correlation length of the random potential [31,59]). At zero temperature it equals the length defined by R_c defined by the Larkin-Ovchinikov length and in general l can be thought of as the Larkin Ovchinikov

length renormalized by temperature. Below this length the elastic manifold sees a smooth potential with well defined derivatives, thus a local random force can be defined. Indeed expanding in u the disorder potential energy in (15) gives a random force term $f.u$ with $f(x) = \sum_K V(x) K exp(-iKx) = \nabla V(R_i)$. In the sum over harmonics the maximum K is $K_{max} = 2\pi/\xi_0$. Thus this expansion is valid only as long as $u \ll \xi_0$. This defines the range of validity of the Larkin regime, i.e at $T = 0$ $x < R_c$ and more generally $x < l$. Of course this first regime only exists if $R_c > a$. From the point of view of the variational solution, this regime corresponds to a replica symmetric part, consistent with the fact that there are no metastable states (and thus no pinning in Larkin's random force model).

ii) For $l_T^2 \ll \tilde{B}(r) \leq a^2$, $\tilde{B}(r) \sim r^{2\nu}$ where $\nu \sim 1/6$: this is the random manifold regime mentioned above where each line sees effectively an independent random potential. This can be seen, on a more mathematical level, from our model by summing over all the harmonics for instance on the replicated Hamiltonian (21). One gets $V(u) \sim \sum_{R_i} \delta(u_a - u_b - R_i)$. For $u \ll a$ only the $R = 0$ term contributes and each line sees an independent random potential. This intermediate random manifold regime holds up to the length R_a such that $\tilde{B}(R_a) \approx a^2$ at which periodicity becomes important. It is noteworthy that for models for which only one harmonic exists, such as for CDW, the random manifold regime does not exist, and one directly crossovers from the Larkin regime to the asymptotic regime iii). In the RM regime replica symmetry is fully broken, a signature of the various metastable states and of pinning.

iii) For $r > R_a$, the periodicity of the lattice becomes important and $\tilde{B}(r) \sim A_d \log|r|$ where A_d is a universal amplitude depending on dimension only. To check the result of the variational method we also computed A_d using a functional renormalization group procedure [56], in a $\epsilon = 4-d$ expansion. These two rather different methods agree at order ϵ within 10%. Within the Gaussian approach, (8) gives the translational correlation function $C(r)$ which has a slow algebraic decay in $d > 2$, $C(r) \sim (1/r)^{A_d}$ and quasi-long range order persists. This is a reasonable lower bound for $C_K(r)$. It may give the exact asymptotic decay or it is also possible that atypical "return to the origin" events (i.e a singularity at $u = 0$ of the scaled probability of u) could make this decay *slower*. A similar situation is discussed in [60].

The above regimes describe *generically* a disordered periodic elastic system. A summary is shown in figure 2 with the main characteristic scales. Quite unexpectedly *periodicity* helps the system to keep quasi-long range translational order ! R_a is not a sort of crystallite size beyond which translational order would be lost, but on the contrary the length above which displacements grow *more slowly*. The difference between these two physical pictures is illustrated in figure 2. This near saturation of the displacements can be understood by noting that the system wants to minimize its total energy due to disorder. A single line thus cares little in making displacements much larger than a since it would merely try to "steal" the random potential of one of its neighbors, with little gain for the total energy and at a huge elastic cost. Periodicity has thus drastic consequences and paradoxically leads to a more ordered situation than for a simple manifold.

Using the variational methods, many other results can be derived ranging from the full crossover function $\tilde{B}(x)$ to the detailed behavior of the translational correlation function $C_K(r)$ and we refer the reader to [48,49] for further details.

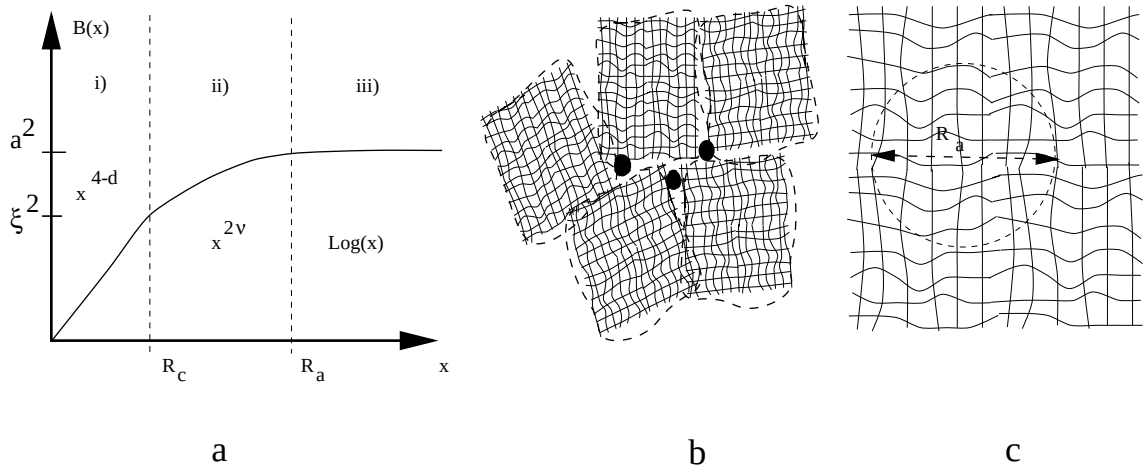

Figure 2: a) The generic behavior of the relative displacements correlation function $B(x)$ for any disordered elastic system, with the three regimes described in the text. As explained in the text depending on the values of R_c or R_a regime i) or ii) might be absent. At large distance displacement grows only logarithmically and quasi-long range translational order exists. b) shows the *incorrect* but commonly believed view of a disordered elastic system. R_a would correspond to a length above which translational order is destroyed and the system would break into "cristallites". Topological defects (represented as black dots) would be generated due to disorder. c) is the *real* situation for weak disorder (the Bragg Glass): R_a is just the crossover scale for which relative displacements are of order a, and above which they grow very slowly. So although displacements can become large, the system preserves quasi-long range translational order, and no topological defect exists.

Dislocations or no dislocations

This striking result that quasi-long range order survives has been derived within an elastic theory, assuming the absence of dislocations. The alleged importance of dislocations in a disordered system[32] makes it mandatory to further investigate carefully whether dislocations can modify the above result. In fact, using an energy argument[49], one can get convinced that dislocations are much less relevant than commonly assumed. The argument is as follows: the core energy of a dislocation loop of length L grows as L^{d-2}. Since a dislocation involves $1/r$ displacements around its core, the total cost associated with an unbound dislocation is therefore $L^{d-2}\ln(L)$. Now the cost of an elastic deformation which can be typically relaxed by allowing for a dislocation loop is, provided $u \sim L^{\nu}$ of the order of $L^{d-2+2\nu}$. So if translational order is destroyed ($\nu > 0$), i.e. if the Larkin or the random manifold regime were true up to infinite scales, it would indeed be favorable to create dislocations. However if quasi long range order persists $\nu = 0$ and for weak disorder, the cost of a dislocation would always be higher for weak disorder than the one of an elastic deformation, and dislocations are *not* generated by disorder. The elastic solution is thus *self consistently stable*. This implies self-consistently the existence of a *thermodynamic glass phase*, as far as energy and very low current transport properties are concerned, retaining a nearly perfect (i.e. algebraic) translational order. Since this phase exhibits Bragg peaks very much like a perfect lattice we christened it the "Bragg glass".

The prediction [49] that a phase *without* topological defects should be stable at weak

disorder, which also applies to the random field XY model, received subsequent further support both from numerical simulations [61,62], analytical calculations in a layered geometry [63,64] and, very recently, a proof using improved scaling and energy arguments [65].

Bragg glass and other glasses

Because of their original periodicity, periodic systems in a random potential have a radically different physics than originally expected: quasi-long range order and no topological defects ! Of course, since it retains a "lattice" structure and Bragg peaks, this glass phase is widely differs from the vortex glass picture based on a random gauge model. It is also different from the naive original picture based simply on elastic manifolds. Indeed, as one sees on on figure 2 if the random manifold regime had survived at large scales (beyond R_a) the same energy argument implies that dislocations *would have been generated* spontaneously (since there $\nu > 0$). The theory of the Bragg glass is thus poles appart from the previously proposed theories for the glass phases of superconductors. Although we have insisted here on the vortex aspects of this phase relevant mostly to vortex systems let us emphasize again that it is quite generic to any elastic disordered system and should be observable in other situations. In particular, CDW systems for which direct measurements of C_K is possible by x-ray diffraction should be good candidates.

For the vortex problem the Bragg glass has of course many experimentally observable consequences. In particular, since such a phase is nearly as good as a perfect lattice as far as translational order is concerned, it is natural to expect it to melt through a first order phase transition. We proposed [49] that the phase seen experimentally at low fields in type II superconductors was in fact the Bragg glass, solving the apparent impossibility of a pinned solid. This allowed to account naturally for the first-order transition and the decoration experiments. Neutron experiments (measuring directly C_K) can be naturally interpreted in term of the Bragg glass [51,52]. For a detailed discussion of the various experimental consequences we refer the reader to [49,66].

But one of the most interesting consequences is provided by the constraints that the mere *existence* of a Bragg glass phase imposes on the $H - T$ phase diagram of superconductors. Indeed since the Bragg glass should *not* contain unbound topological defects, a phase transition should exist towards *another* phase containing topological defects when disorder is increased (an upper bound for the limit of stability for the Bragg glass is of course $R_a \sim a$). One can show that in the range of fields relevant for most high T_c superconductors, increasing the field is equivalent to increasing the disorder. Thus the simple existence of the Bragg glass imposes [49] that a transition in field should exist. The nature of the high field phase is still unclear both experimentally and theoretically. It could be either a pinned liquid or another glass (the putative "vortex glass" ?). What is clear is that due to the presence of topological defects in that other phase one expects it to melt in a much more continuous fashion into the liquid (or simply to undergo a crossover), and thus may be consistent with the observed continuous (or gradual) transition at high fields. These considerations led us to propose the phase diagram depicted schematically in figure 3. Let us emphasize again that the existence of such a transition in field is the direct consequence of the fact that the low field *thermodynamic* phase has no topological defects.

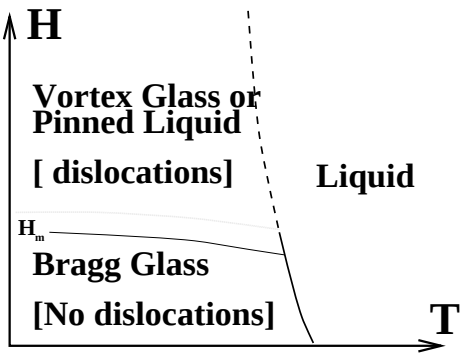

Figure 3: The stability region of the Bragg glass phase in the magnetic field H, temperature T plane is shown schematically. The thick line is expected to be first order, whereas the dotted line should be either second order or a crossover. Upon increasing disorder the field induced melting occurs for lower fields as indicated by the thin solid line.

Several recent experiments can be interpreted to confirm the picture proposed in [48,49]. In BSCCO neutron diffraction peaks are observed at low fields and disappear upon raising the field [67]. The phase diagram of BSCCO [28] is also compatible with our theory, the second magnetization peak line corresponding to the predicted field driven transition. This line is found to be relatively temperature independent at lower temperatures and to be shifted downwards upon increase of point disorder [68,69]. Similar types of phase diagrams are also observed in a variety of materials, including YBCO, organic superconductors and heavy fermion compounds, which seem to indicate that this is indeed a quite generic mechanism. More experimental consequences and references can be found in [66]. More experimental work will be needed to confirm the proposed picture.

On a theoretical level, going beyond the simple topology of the phase diagram proves for the moment to be very difficult, since we are impaired by our lack of knowledge of the high field phase. Describing such a phase would mean to be able to treat a disordered elastic system in which there is also a finite amount of topological defect, a quite challenging problem, but at present beyond our reach. We even lack tools to obtain semi-quantitative estimates for the positions of the melting lines. A blind application of a Lindemann criterion to the transition in a field gives a numerical estimate [66] of the melting field H_M for BiSCCO of $H_M \sim 400G$ in good agreement with the observed experimental values [68]. However it is far from clear that the Lindemann criterion which works indeed quite well to describe thermal melting, can also capture the physics of this peculiar disorder-induced melting transition. It could however give correctly the value of the field at the transition as can be checked for special geometries where the field transition can be computed. Clearly, dealing with a disordered elastic system in presence of a *finite* density of topological defects (the only way to really describe either the melting transition or the transition in field), is the next challenge !

3.5 The peculiar case of $d = 2$

The Bragg glass is thus the prototype of an elastic glass phase with internal periodicity. In its physics $d = 2$ plays a particular role because thermal fluctuations become important. Also $d = 2$ applies to a variety of experimental situations such as magnetic bubbles, charge density waves, colloids, random XY model. In addition, further analytical methods become available [33,70,71,46] to analyze the problem with and without dislocations.

When dislocations are *excluded* by hand a glass phase is believed to exist for $T < T_g$ in $d = 2$. For the simpler $N = 1$ component model (i.e the random field XY model) this was shown by Cardy and Ostlund[33] (CO) who used n replicated coupled XY models, mapped them onto a Coulomb gas with $n(n-1)/2$ types of vector charges and constructed the RG equations. They set $n \to 0$ implicitly assuming a replica symmetric fixed point. The resulting RG equations, valid near T_g, possess a non-trivial perturbative fixed point for $T < T_g$ at weak disorder $g = g^* \propto T_g - T$. CO concluded that a "glass" phase exists, controled by this fixed point. In this phase, one coupling constant flows to infinity, a rather peculiar feature. This $N = 1$ model is known to describe flux lines lying in a plane (a system where topological defects are indeed excluded by construction). These results and their connections to glass phases of flux lines were extended in [46,72,73,74,75], and the disorder averaged correlation function $\tilde{B}(x) = \overline{\langle u(x) - u(0)\rangle^2}$ was found to grow as $\tilde{B}(x) \sim A(\log|x|)^2$, faster than $\tilde{B}(x) \sim T \log|x|$ which holds in the high temperature phase and for the pure system.

By analogy it was argued [49] that similar results hold in the case of the triangular lattice. This was verified explicitly recently [74] using the RG on the fully coupled $N = 2$ component model required to describe a triangular lattice. The calculation leads to a glass phase for $T < T_g$ described by *a plane* of perturbative fixed points. The growth of relative displacements is found to be asymptotically isotropic with $u_T^2 \sim u_L^2 \sim A_1 \ln^2 r$, with universal subdominant anisotropy $u_T^2 - u_L^2 \sim A_2 \ln r$. where A_1 and A_2 depend continuously on temperature and the Poisson ratio σ. The fixed points obtained in [33] and in [74] are thus the natural continuation to $d = 2$ of the non trivial fixed point which describes the Bragg glass phase for $d = 3$ (and $N = 1$, $N = 2$ respectively). $d = 2$ thus appears as the lower critical dimension of this fixed point. While for $d \geq 2$ the Bragg glass fixed point is a *zero temperature fixed point* where temperature is (dangerously) irrelevant, in $d = 2$ the glass phase is described by a *line of fixed points* where temperature is *marginal*. Since entropy still plays a role in these $d = 2$ periodic glasses they can be called *marginal glasses*. This is illustrated in their dynamics: the dynamical exponent z (such that $t \sim x^z$) was computed below T_g (assuming equilibrium dynamics, equivalent to assuming replica symmetry in the statics - see below) for $N = 1$[70,76] and for the triangular lattice[74] $N = 2$:

$$z - 2 \sim 2e^{\gamma}\tau \qquad \text{CO model} \tag{25}$$

$$z - 2 \sim 3e^{\gamma}\tau \frac{(2+\alpha)(\frac{2-\alpha}{2+\alpha})^{\frac{2-\alpha}{4}}}{2I_0(\alpha/2) - I_0(\alpha)} \qquad \text{triangular lattice} \tag{26}$$

It is also continuously varying with the reduced temperature $\tau = (T_g - T)/T_g$ and the Poisson ratio σ through $\alpha = 2(1+\sigma)/(3-\sigma)$. γ is the Euler constant and I_0 the modified

Bessel function). This finite z dynamical exponent yields the following I-V (equivalently v-f) characteristics:

$$v \sim \mu_0 f \left(\frac{f}{f_c} \right)^{\frac{z-2}{2}} \tag{27}$$

Note that this is different from the creep law (20) valid in $d > 2$ which is typical of a $T = 0$ fixed point with a dynamical exponent formally $z = \infty$. The $d = 2$ result (27) can be interpreted as barriers which increase logarithmically $U(j) \sim \ln(1/j)$ with decreasing current rather than algebraically as in $d > 2$.

There are also some interesting issues related to the possibility of replica symmetry breaking in this phase. The Gaussian replica variational method (GVM) described above, when applied to this model, leads to a one step replica symmetry breaking (RSB) solution below T_g and thus correctly predicts the transition but yields mean squared relative displacements growing as $u^2 \sim T_g \ln r$. This is a different result from the replica symmetric (RS) RG prediction $u^2 \sim A_1 \ln^2 r$. The GVM, being by construction an approximation, neglects some non-linearities and has no a priori reason to yield the exact result. However it may be a hint that, if allowed, RSB will occur in this model. It was indeed shown within the RG [77,78] that the Cardy Ostlund RS-RG flow is unstable to an infinitesimal RSB perturbation at and below T_g. The issue was thus raised [77] of whether the RS-RG may miss some of the physics related to RSB. The numerical studies presently available show discrepancies [79,80], and their analysis is not yet fully satisfactory. Though there is a more recent trend [81,82,83,84] towards a behavior consistent with the RSRG, it is still only qualitative agreement. Since a more careful treatment of the effects of RSB may reveal that deviations from the RS-RG result are small [85], e.g only in the amplitude of the $\log^2 r$ [77], more precise numerical tests should be performed.

When topological defects are *allowed* the above picture will probably be modified. Indeed it was shown in [33,70] that for the $N = 1$ component model these defects are perturbatively relevant near T_g. As argued in [49] T_g for the triangular lattice is well above the KTNHY melting temperature T_m and dislocations should then be relevant near T_g for the $N = 2$ triangular lattice as well. At low temperature however, much less is known about the importance of dislocations. The common belief [2], which is by no means rigorously established, is that if dislocations are allowed, no true glass phase will exist at $T > 0$ in $d = 2$. In the simpler random phase shift model (which does capture some of the physics of the full problem), a high temperature phase with unbound dislocations was found to be *reentrant* at low temperatures in [86,87] suggesting the importance of topological defects at low temperature. It was pointed out in [49], from a study of the CO RG flow, that at low temperature the scale at which the lattice is effectively dislocation-free (i.e the distance between unpaired dislocations) can be *much larger* than the translational length R_a. Thus even in $d = 2$ the Bragg glass fixed point may be useful to describe the physics, as a very long crossover or maybe directly at $T = 0$. It was also pointed out in [88] that the conventional CO RG will not be adequate at low temperature since it assumes a *thermalized description* of the vortices, neglects important effects such as the pinning of dislocations by disorder and the position dependence of their fugacity. A similar idea was recently proposed and pushed

further by Nattermann et al. [89] who reconsidered the simpler random phase shift model. They explicitly showed that [86] was incorrect at low temperature and proposed a modified approach which leads to a phase which is *defect free* at low temperature. Though these new approaches need to be put on a firmer theoretical footing (there are several underlying assumptions) and though it is still an open question how they carry to more complicated and realistic elastic models, it is remarkable that the results of [89] do provide another non trivial example, besides the $d = 3$ Bragg glass, of a case where topological defects are *less relevant* than is naively assumed.

Thus the question of dislocation is subtle even in $d = 2$. Using naively the RG one would conclude that there are always dislocations. This is intuitively clear since in $d = 2$ dislocations are simply point like defects and thus much easier to create by disorder. But it could also be too naive, since disorder obviously modifies also the interactions between the defects which are mediated by the elastic interactions. Thus it is still an open question whether this is really correct. In any case, even if it was, the length between unpaired dislocations R_D clearly grows *much faster* than R_a (it can be estimated as $R_D \sim R_a e^{\ln^{1/2}(R_a)}$ see [49]). By reducing the disorder and temperature one can thus, even in $d = 2$, have arbitrarily large dislocation free regions where the main source of translational order decay is from elastic deformations. Regimes where $R_D \gg R_a$ have indeed been seen in magnetic bubble experiments (see discussion in [49]).

Let us conclude this section by noting the amusing twist by which the CO model, on which the proposal of the "vortex glass" phase [30] was based originally, has turned out, upon further analysis, to be of a totally different nature. Indeed the CO glass phase is rather the continuation to $d = 2$ of the topologically ordered Bragg glass phase which we have argued exists as a thermodynamic stable phase in $d = 3$.

4 Dynamics of driven disordered lattices

Obtaining a quantitative description of the dynamics of driven interacting systems with disorder is the next challenging problem. Beyond vortex lattices it is also important for several other experimental systems such as Wigner crystals [41] moving under an applied voltage, lattices of magnetic bubbles [43] moving under an applied magnetic field gradient, Charge Density Waves (CDW) [40] and colloids [90] submitted to an electric field, driven Josephson junction arrays. For many of these systems transport measurements are a useful way to probe the physics of the system, and sometimes the only way if direct imaging cannot be performed. Dynamic properties have thus been studied for some time, especially for the case of CDW or for driven manifolds and their relation to growth processes[91] described by the Kardar Parisi Zhang (KPZ) equation[92,1], using a variety of methods that we will only briefly review here. Curiously the similar problem of a periodic *lattice* (with additional periodicity transverse to the direction of motion) was not scrutinized until very recently, maybe because it was naively thought that it falls in the same class as the above problems. Fortunately, as for the statics, (transverse) periodicity drives again the system to a novel behavior, the richness of which is far from being understood. We will thus mainly devote the rest of this section to this particular case of dynamical problems.

To tackle the dynamics of such periodic systems, it is important to know whether topological defects in the structure are generated by disorder, temperature and the driving force or their combined effect. Indeed if such defects exist, the flow will not be elastic, but will turn into the so-called plastic flow, with a radically different behavior. It is an important and still largely open question to determine when plastic rather than elastic motion occurs but quite generally one expects plastic motion for strong disorder situations, at high temperature, and probably close to the threshold in low dimension (for CDW see e.g.[93]). This is confirmed by experiments and numerical simulations. Indeed, there has been a large number of studies on plastic flow [94], e.g in the context of superconductors where a H-T phase diagram with elastic flow regions and plastic flow was observed[95]. Several experimental new effects have been attributed to it such as the peak effect [96,95], unusual broadband noise [97] and fingerprint phenomena in the I-V curve [98,99,100]. Close to the threshold and in strong disorder situations the depinning is known to proceed [101] through filamentary flow in what can be called "plastic channels" [102,103] between pinned regions. Despite numerous studies, mostly numerical ones, a detailed theoretical understanding [104] of this regime is still sketchy.

One could expect to be in a better position to attack the problem of the elastic flow. At first the task seems formidable. Experience from other glassy systems, such as spin glasses, has taught us to expect an extremely complicated dynamics due to the peculiar features of the energy landscapes [105]. Generally three main dynamical regimes can be established. Far below the threshold the system can move only through thermal activation. This is the creep regime where qualitative arguments have been developed [31]. One would like to check whether these rather phenomenological arguments can be confirmed by more direct (and hopefully rigorous) dynamical calculations. The second regime, near the elastic depinning transition, has been intensely investigated in similarity with standard critical phenomena (see e.g [106,107,108]) where the velocity plays the role of an order parameter. The third regime, which is the one we will concentrate on here, is far above the threshold. In this regime things may look more rosy, since one could also imagine that a sliding system averages in fact enough over disorder, to recover a simple enough behavior, in fact much simpler than in the statics. Indeed it was observed experimentally some time ago in neutron diffraction experiments [109], and in more details recently [51] that at large velocity the vortex lattice is more translationally ordered than at low velocity. In this regime, since the velocity is large, one is not so much interested in the v-F characteristic, but much more on the positional properties of the moving system. A question of prime interest is thus whether at large enough velocity glassy effects disappear and whether one recovers a perfect lattice.

Before concentrating on this issue and seeing that the answer crucially depends on the periodic nature of the driven system, let us look at the general behavior of driven lattices and first establish the equation of motion.

4.1 The basic equation

The conventional description for the dynamics of these systems is in terms of overdamped dynamics with a microscopic friction coefficient η. Let us denote by $R_i(t)$ the true position of an individual particle in the laboratory frame and assume that the lattice as a whole moves with a velocity v. We thus introduce the displacements $R_i(t) = R_i^0 + vt + u_i(t)$ where

the R_i^0 denote the equilibrium positions in the perfect lattice with no disorder. u_i represent the displacements compared to a moving perfect lattice (and corresponds to the position of the i-th particle in the moving frame). The definition of v imposes $\sum_i u_i(t) = 0$ at all times. For a manifold i would be a continuous index spanning the internal dimension of the manifold. Using these variables the equation of motion can be written in the *laboratory frame*:

$$\eta \frac{du_i(t)}{dt} = -\frac{\delta H_{el}}{\delta u_i} + \int_r \partial V(r)\delta(r - R_i^0 + vt + u_i(t)) + f - \eta v + \zeta(R_i(t), t) \tag{28}$$

where f the external uniform force driving the system and ζ is a thermal noise which satisfies $\overline{\zeta(r,t)\zeta(r',t)} = 2T\eta\delta(r - r')\delta(t - t')$. The other two forces acting on the system are the elastic force, derivative of the elastic Hamiltonian $H_{\rm el}$ (3), and the pinning force, coming from the coupling (9) to the random potential. As for the statics it is fruitful to take the continuous limit. Using the decomposition of the density (14) allows to rewrite (28) in the simple form

$$\eta \partial_t u_{rt}^\alpha + \eta v \cdot \nabla u_{rt}^\alpha = \int_{r'} \Phi_{\alpha\beta}(r - r') u_{r't}^\beta + F_{\rm pin}^\alpha(r,t) + f_\alpha - \eta v_\alpha + \zeta_\alpha \tag{29}$$

where $\Phi_{\alpha\beta}(r - r')$ is the elastic matrix. The convection term $\eta v \cdot \nabla u_\alpha$ comes from the standard Euler representation when expressing the displacement field in the laboratory frame. Note that this term is *not* the gradient of a potential, as a consequence of the fact that the system is out of equilibrium with energy constantly injected and dissipated. The pinning force is given by:

$$F_\alpha^{\rm pin}(r,t) = -\delta H_{\rm pin}/\delta u_\alpha(r,t) = V(r)\rho_0 \sum_K iK_\alpha \exp(iK \cdot (r - vt - u(r,t))) \tag{30}$$

(as for the statics we only write the important Fourier components). A manifold, lacking the periodicity, can also be described by (30) simply by letting the discrete sum over the reciprocal lattice vectors K become an integral $\int dK$ to reproduce the δ function of the density on the manifold.

(29-30) is the complete equation one would have to solve, and again one is faced with the nonlinearities in (30): they both prevent one from solving the equation and of course lead to most of the interesting effects.

4.2 Critical force and large v expansion

To tackle these formidable equations (29-30), the first angle of attack, again pioneered by Larkin [110], and by Schmidt and Hauger [111], is to perform a large velocity expansion of (29). Indeed at large v, (30) oscillates rapidly due to the terms in Kvt and vanishes[a]. One can then compute the displacements u in an expansion in $1/v$. Solving at first order and using

[a] At that point the astute reader will have noticed some forthcoming problems from the modes such that $K \cdot v = 0$. We will come back to that point later.

the corresponding expression of u, (30) gives a correction to the average velocity [110,111]

$$\delta v_\alpha = -\frac{1}{2}\sum_K \sum_{I=L,T} \int_{BZ} \frac{dq}{(2\pi)^d} K_\alpha (K.P^I(q).K)\Delta_K \frac{v.(K+q)}{\Phi^I(q)^2 + (\eta v.(K+q))^2} \tag{31}$$

where $P^{L,T}(q)$ are the standard longitudinal and transverse projectors and $\Phi^{L,T}(q)$ the elastic energy of longitudinal and transverse modes, e.g in $d=2$ at small q, $\Phi^L(q) = c_{11}q^2$ and $\Phi^L(q) = c_{66}q^2$ where c_{11}, c_{66} are respectively the bulk and shear modulii.

One can push this formula, which is valid only at large v, beyond its domain of validity to estimate[2] in a very qualitative way the value of the threshold field by the criterion $\delta v/v \sim 1$ replacing v by f_c/η. More interestingly, in addition to giving an estimate for f_c which is found to be consistent with the Larkin Ovchinikov arguments of section 3.2, the large v expansion allows in principle to compute the displacement correlation function. This was done in [112] where it was concluded that at low T and above a certain velocity the moving lattice becomes a perfect crystal again at an effective temperature $T' = T + T_{sh}$. The effect of pinning was then described [112] by some effective *shaking temperature* $T_{sh} \sim 1/v^2$ defined by the relation $\langle |u(q)|^2 \rangle = T_{sh}/c_{66}q^2$. The physical picture that emerges from the naive large v expansion seems at first very reasonable since the system in fast motion averages enough over disorder. Since the disorder vanishes and is replaced by "thermal" effects, this approach would suggest bounded displacements in $d > 2$ and absence of glassy properties in the moving solid. At least at large enough velocities, the dynamics would thus seem much more simple than the corresponding statics !

However, the problem is more complicated than it looks and this naive approach is incorrect. For reasons that we will explain in section 4.4, the large v expansion is invalid [113,114] and transverse periodicity leads to (well hidden) divergences in perturbation theory. In addition, due to the driving of the system, other relevant terms, such as random forces are generated. As for the statics, a correct study of the problem requires to fully treat the nonlinearities of (30), and sharper tools than mere perturbation expansion are needed.

4.3 Methods and what follows

Fortunately, more powerful methods exist. The most standard one is to introduce a field theoretical description of (29) which we write in a compact form:

$$(R^{-1})^{\alpha\beta}_{rtr't'} u^\beta_{r't'} = f_\alpha - \eta v_\alpha + f_\alpha(r,t,u_{rt}) \tag{32}$$

To do so, one introduces the Martin-Siggia-Rose-de Dominicis-Janssen generating functional [115] given by

$$Z[h,\hat{h}] = \int DuD\hat{u} e^{-S[u,\hat{u}] + \hat{h}u + ih\hat{u}} \tag{33}$$

where $\hat{h}, h$ are source fields. The MSR action corresponding to the equation of motion (32) is

$$S[u,\hat{u}] = S_0[u,\hat{u}] + S_{int}[u,\hat{u}] \tag{34}$$

with

$$S_0[u,\hat{u}] = \int_{rtr't'} i\hat{u}^\alpha_{rt}(R^{-1})^{\alpha\beta}_{rt,r't'}u^\beta_{r't'} - i\hat{u}^\alpha(f_\alpha - \eta_{\alpha\beta}v_\beta) - \eta T\int_{r,t}(i\hat{u}^\alpha_{rt})(i\hat{u}^\alpha_{rt}) \quad (35)$$

$$S_{int}[u,\hat{u}] = -\frac{1}{2}\int drdtdt'(i\hat{u}^\alpha_{rt})(i\hat{u}^\beta_{rt'})\Delta^{\alpha\beta}(u_{rt} - u_{rt'} + v(t-t')) \quad (36)$$

where we recall that $\Delta(u-u')$ is the disorder correlator. Causality imposes that $R_{rt,r't'} = 0$ for $t' > t$ and the Ito prescription for time discretization implies $R_{rt,r't} = 0$. In such driven problems, space symmetry is broken by the motion and $C_{-r,t} \neq C_{r,t}$ when v is non zero.

This formalism is widely used to study dynamical problems. It has the advantage of treating separately the correlation $C^{\alpha,\beta}_{rt,r't'} = \overline{\langle u^\alpha_{rt}u^\beta_{r't'}\rangle}$ and response functions $R^{\alpha,\beta}_{rt,r't'} = \delta\overline{\langle u^\alpha_{rt}\rangle}/\delta h^\beta_{r't'}$ which measures the linear response to a perturbation applied at a previous time. They are obtained from the above functional as $C^{\alpha\beta}_{rt,r't'} = \langle u^\alpha_{rt}u^\beta_{r't'}\rangle_S$ and $R^{\alpha\beta}_{rt,r't'} = \langle u^\alpha_{rt}i\hat{u}^\beta_{r't'}\rangle_S$ respectively. Although these two functions are usually related by the fluctuation dissipation theorem $TR^{\alpha\beta}_{r,t} = -\theta(t)\partial_t C^{\alpha\beta}_{r,t}$ for equilibrium problem, this does not need to be the case in dynamical ones. The MSR formalism is thus able to tackle out of equilibrium dynamics [116,117] for which the fluctuation dissipation theorem (FDT) does *not* hold. This is the case here we are studying a moving system which does not derive from a Hamiltonian.

Starting from MSR one can either use a dynamical mean-field theory [118,116], or since MSR is a field theoretical formulation derive renormalization group equations by integrating over short scales. Using such a renormalization group one can go beyond the large v expansion and access properties at the depinning transition. Such a procedure was pioneered to study the depinning of manifolds in [108,119,107]. A similar situation to that of the static functional renormalization develops. The disorder correlator $\Delta(u)$ flows to a fixed point function (which corresponds to the threshold fixed point $v = 0$) which is *non analytic*. This non analyticity was shown to be directly related to the critical force by $f_c \sim \Delta'(0^+)$. As can be expected from Larkin's model, the scale at which the non analyticity appears is the Larkin length R_c. This method suitably generalized [120,114] allows also for the study of the moving periodic system.

Within the MSR formalism one can also study the generation of different additional terms in the equation of motion. In particular, as was first observed in growth processes, and later in driven manifolds[92,1], the application of an external driving force breaks the symmetry $u_x \to -u_x$ (as can be seen on (32)). This allows, from pure symmetry considerations, the generation of non linear terms, such as $(\nabla u)^2$. These terms, the so called KPZ terms, may be relevant and change the large scale behavior drastically. The effect of such terms in driven dynamics is a subject of active investigations [1,121]. One can show explicitly that in the present problem, not only these KPZ like terms, but also linear terms which are allowed by symmetry, are indeed generated [114] from a finite cutoff effect. Since their bare value is very small they may be important only at very large length scales. Other non trivial terms can be generated in such dynamical systems. One of the simplest is the random force. It is intrinsically a non equilibrium effect since it cannot exist in the statics by the symmetry $u \to u + a$. Usually the existence of such terms are conjectured [122] since their explicit

calculation from the original equation of motion (29) is difficult. We will come back to that point and give an explicit derivation of this term in the next section.

4.4 Periodicity peculiarities: the Moving glass

We now come back to the detailed physical properties of the periodic structures. The physics becomes transparent when looking at the pinning force (30): all the modes of the disorder such that $K \cdot v = 0$ do *not* have a direct time dependence (some time dependence can be introduced through the displacement field u), and will *not* average out even at large velocity. This is assuming that motion occurs along one of the lattice direction, which is physically reasonable. In such case the large v expansion breaks down ! The equation of motion thus contains a static disorder component perpendicular to the direction of motion.

$$F_\alpha^{\rm stat}(r,u) = V(r)\rho_0 \sum_{K.v=0} iK_\alpha \exp(iK\cdot(r-u)) \tag{37}$$

Since this force is perpendicular to the velocity, it has no impact on the velocity correction itself, for which the large v expansion is still perfectly valid. On the other hand the correlation functions of the displacements will be drastically affected.

Again the periodic system proves its peculiar nature. Note that here periodicity *perpendicular* to the direction of motion is the crucial ingredient. The effects described here will thus be absent for systems for which such transverse periodicity does not exist such as single q CDW (for which the density is only modulated in one direction[40]), or manifolds pulled orthogonally to their direction, such as in growth phenomena. On the other hand systems such as vortex systems or the Wigner crystal are directly affected.

Beads on a string

The resulting physics can be easily understood by focusing on the component of the displacement u affected by this remaining static disorder. Although this is an approximation it provides a clear enough view of the physics and can be confirmed by a more rigorous renormalization calculation[114]. Clearly only transverse displacements are affected, and the static equation is

$$\eta\partial_t u^y_{rt} + \eta v\cdot\nabla_x u^y_{rt} = c\nabla^2 u^y_{rt} + V(r)\rho_0 \sum_{K.v=0} iK_y \exp(iK\cdot(r-u)) \tag{38}$$

where we denote generically by y the transverse directions and we have chosen isotropic elasticity for simplicity. If it were not for the linear term $v\nabla_x u$, this equation would be identical to the one describing the relaxational dynamics of a periodic manifold in a random potential (without driving). The linear term is the one taking the dissipation into account and making the problem different from an Hamiltonian one. From the point of view of the solution however this term merely introduces a different scaling between the direction of motion (x) and the perpendicular directions (y), $L_x \sim L_y^2$. Due to the presence of the static disorder, one thus expects the transverse components to present all the characteristics of a static disordered elastic system and thus to exhibit pinning, to have unbounded growth of

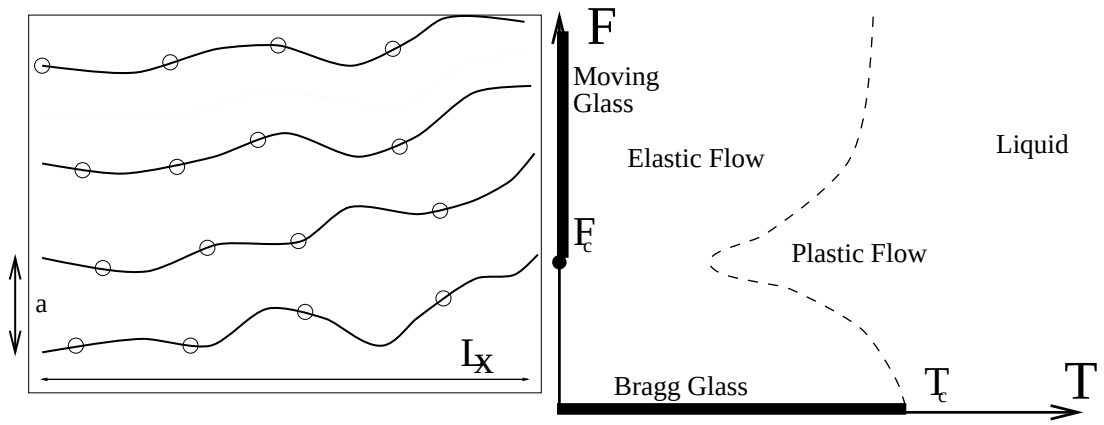

Figure 4: a) Motion in the moving glass occurs through static channels wandering at distance a over lengths $L_x \sim L_y^2$. If dislocations are present ($d = 2$ or strong disorder in $d = 3$) they should lead to a decoupling of channels, as indicated by the dotted line. A square lattice was represented for simplicity. b) Phase diagram in force F, temperature T, for weak disorder and in $d = 3$. At zero external force the system is in the free Bragg glass state. At large velocities, in the moving Bragg glass one. This suggests that in this case the depinning transition could be purely elastic.

displacements etc. In short to be a glass. This time the periodicity makes the system more disordered than was naively expected !

Of course there are various important differences with the static problem. The simplest one comes from the scaling $L_x \sim L_y^2$, which makes the disorder in the moving system only relevant for $d \leq 3$. For $d > 3$ the moving system is not a glass but a perfect crystal at weak enough disorder or large velocities. Apart from this rescaling the physical properties of the moving system presents some similarities with the one of the elastic disordered systems exposed in section 3.4. In particular there will be a *static* solution for the transverse displacement, becoming rough at a length scale $Rx \sim R_y^2$, analogous to the R_a of the static problem. Estimates a la Fukuyama-Lee similar to the one of section 3.3 give:

$$R_y^a \sim (a^2 vc/\Delta)^{1/(3-d)}, \qquad R_x^a = v(R_y^a)^2/c \tag{39}$$

The moving glass is highly anisotropic since R_x^a/R_y^a diverges as $v \to \infty$.

Thus, the moving vortex configurations can be described in terms of *static channels* that are the easiest paths where particles follow each other in their motion like beads on a string[113,114]. Channels in the elastic flow regime are fundamentally different in nature from the one introduced to describe slow plastic motion between pinned islands [102,103]. In the moving glass they form a manifold of elastically coupled, almost parallel lines or sheets (for vortex lines in $d = 3$) directed along x and characterized by some transverse wandering u_y. In the laboratory frame they are determined by the static disorder and do not fluctuate with time. In the moving frame, since each particle is tied to a given channel which is now moving, it indeed wiggles and dissipates but the motion is highly correlated with the neighbors. An image of this channel picture is shown in figure 4. Such channels were subsequently observed in numerical simulations [123] and in recent decoration in motion experiments [124]. The physical picture emerging is thus completely different from the image

of a perfect lattice submitted to an extra shaking temperature. This concept may be correct however in the liquid.

Determining the roughness of the channels is not an easy task. It is of course impossible to directly borrow the static result since the problem is by nature dynamics because of the dissipation term $v\nabla_x u$, and as we saw in section 4.3, many terms can be generated due to the motion. The full solution of the problem is not known, and in particular the effects of the nonlinear KPZ-like terms. If one ignores such terms, it is possible to apply the FRG to extract the roughness of the channels. We quote here only the RG equation for the disorder term, for the periodic structure at $T = 0$ (for the complete calculation see [114])

$$\frac{d\Delta(u)}{dl} = \Delta(u) + \Delta''(u)(\Delta(0) - \Delta(u)) \tag{40}$$

where a factor $\frac{1}{4\pi vc}\epsilon$, with $\epsilon = 3 - d$ has been absorbed in $\Delta(u)$ (chosen to be of period 1). For $d > 3$ disorder renormalizes to zero and the moving system is a crystal. For $d < 3$ Δ flows to a new fixed point $\Delta^*(u) = \Delta(0)(l) + u^2/2 - u/2$, showing that the static disorder is still relevant in the moving structure (with the same conclusion[114] in $d = 3$). This new fixed point describes the moving glass phase at $T = 0$. The value of $\Delta(0)(l)$ grows unboundedly as $\Delta(0)(l) = \Delta(0)e^{\epsilon l}$ which indicates the existence of a random force along the y direction, generated under renormalization. A similar force is generated along x [114]. Thanks to the RG formulation we are able to *explicitly* compute this random force

$$\delta\Delta_0^{\alpha\beta} = \sum_K \int_q K^4 K_\alpha K_\beta \Delta_K^2 \frac{\eta^2 v^2 (K_x + q_x)^2}{(c^2 q^4 + \eta^2 v^2 (K_x + q_x)^2)^2} \tag{41}$$

where Δ_0 is the correlator of the random force. The divergences of $\Delta(0)$ does not spoil the above fixed point, since one can always separate the random force $\Delta(0)$ and the non linear part $\Delta(u) - \Delta(0)$. The generated random forces will have very different impact depending on the dimension. In $d = 3$ displacements only grow logarithmically, so the MG conserves quasi-long range translational order. Thus similarly to the statics, the MG in $d = 3$ at weak disorder or large velocity is expected to retain perfect topological order. In that case one would go from a static Bragg glass without dislocations to a moving Bragg glass also without dislocations (at large velocities). It is thus possible that in $d = 3$ the depinning occurs without an intermediate plastic region, leading to the phase diagram of figure 4.

In $d = 2$ however displacements grow algebraically and dislocations are more likely to appear. The existence of channels[113] then *naturally* suggests a scenario by which dislocations affect the MG: when the periodicity along x is retained, e.g., presumably in $d = 3$ at weak disorder, the channels are coupled along x. Upon increasing disorder or decreasing velocity in $d = 3$, or in $d = 2$, decoupling between channels can occur, reminiscent of static decoupling in a layered geometry [64] (see also [125]). Dislocations are then inserted between the layers, naturally leading to a flowing smectic glassy state, recently observed in $d = 2$ numerical simulations [123]. Indeed, the transverse smectic order is likely to be more stable than topological order along x, because of particle conservation [114].

The problem of the behavior of dislocations in the moving glass system is of course still open, and constitutes as for the statics one of the most important issues to understand.

Transverse pinning

As an important consequence of the existence of the MG, barriers for transverse motion exist once the pattern of channels is established. Thus the response to an additional small transverse force F_y is very non linear with activated behavior and hysteretic behavior (history dependence). At $T = 0$ and neglecting the dynamic part of the disorder a true transverse critical current J_y^c exists. This can be seen by adding a transverse force in (29) J_y^c can then be estimated by balancing the pinning energy with the transverse Lorentz force acting on a Larkin domain:

$$J_y^c = \frac{c}{\phi_0 r_f} \Delta^{1/2} (R_y^c)^{-(d-1)/2} (R_x^c)^{-1/2} \sim \widetilde{\Delta}^{2/(3-d)}$$

where $\tilde{\Delta} = \Delta/v$ is an effective velocity-dependent disorder. A more rigorous derivation can be obtained from the RG equation (40). Since its fixed point has a *non analyticity* at $u = 0$, (leading to $\Delta'(0^+) = 1/2$) there is a critical force, determined at the Larkin length L_y:

$$F_c = \int dq G(q) \Delta'(0^+) = \frac{\epsilon}{4\pi v c_y} \sim (R_y^c)^{-2} \tag{42}$$

(42) coincides with the above more qualitative derivation. In $d = 3$ one finds:

$$R_c^y \sim e^{\frac{4\pi v c_y}{\Delta_0}} \tag{43}$$

The MG is thus dominated by the competition between the random force and the critical force.

The transverse critical force is a subtle effect since it apparently breaks the rotational symmetry of the problem. In fact it is a purely dynamical effect due to barriers preventing the system to reorient the channels in the direction of the total applied force. It does *not* exist for a single driven vortex line (or for any manifold driven perpendicular to itself) in a random potential, except if the potential is sufficiently correlated in the direction of motion (such as a periodic potential). In some sense here the elasticity of the manifold provides the necessary correlations.

After the prediction of the moving glass, the transverse critical force at $T = 0$ in $d = 2$ was observed in numerical simulations[123,126] (see also [127,128]) and found to be a fraction of the longitudinal critical force. These predictions can be also tested in experiments on the vortex lattice, or other systems such as colloids, magnetic bubbles or CDW, or or numerical simulations. Additional physical consequences and references can be found in [113,114].

4.5 Dissipative glasses

The physics of a periodic moving system has thus some novel properties and in particular was shown to preserve glassy effects (at least for $T = 0$) even in an out of equilibrium regime where the dynamics is *non potential*. We have proposed this physical system as a first example of a "dissipative glass", i.e. a glass with a constant dissipation rate in the stationary state. It seems to hint that non potential dynamics can indeed exhibit glassy properties, a question which one can ask in a more general context.

It was asked recently [129] in the context of spin systems. There it was also found, within mean field, that some glassy properties survive, a conclusion which was going against previous conclusions in mean field models. In finite dimensional system (finite N) we expect these non potential glassy effects to be even stronger and even in some cases survive at finite temperature [114,130]. With hindsight one could in fact consider that a prototype of these systems, even if oversimplified, is the example of a particle diffusing in a random non potential flow with long range correlations, a problem studied a long time ago [131] using a RG approach (it was also solved recently in the large N limit [132]). Remarkably this problem already exhibits glassy effects and a finite temperature fixed point. This prototype model, and more generally all non potential problems (including e.g the moving glass) are described by a Fokker Planck operator whose spectrum is not necessarily real (by contrast with potential problems which are purely relaxational). We also want to point out the deep and interesting connection [114] that exists between the new type of dissipative glassy problems described here and the study of general *non hermitian* random operators. These operators, which in their low dimensional versions can be termed *non hermitian quantum mechanics*, appear in several problems recently studied with a renewed interest (such as vortex lines with tilted columnar defects [18,121], spin relaxation in random magnetic fields [60] or again diffusion of particles and polymers in random flows [133,131,130]). Exploring this connection further, as well as the question of the classification of these glasses and the study of their physical properties is still a largely open but extremely challenging field.

5 Conclusion

We have examined some aspects of the statics and dynamics of disordered elastic systems in presence of disorder, both for manifolds and periodic systems. Although these two classes of systems share many basic properties, periodic structures exhibit a new type of physics quite different and unexpected compared to elastic manifolds. Indeed, for static properties, periodicity helps the system to resist to the disorder and to preserve quasi long range order, while still having the energy landscape and many metastable states of a glass. In turn this preservation of quasi-translational order leaves the system quite stable to the proliferation of topological defects such as dislocations, much in the spirit of an honest solid. This two-faced state (the Bragg glass) both a lattice and a glass seems to be realized in vortex systems, and to explain many of the observed experimental features. The mere existence of the Bragg glass, having *no* unbound dislocations at equilibrium, implies the existence of a transition upon increasing the disorder (or equivalently the magnetic field). This suggests that two glass phases might exist in vortex systems, that could be distinguished by the presence or absence of free topological defects. The existence of such a transition and of the Bragg glass cannot be anticipated by looking at the physics of manifold alone, for which disorder always wins leading to rough and defective ground states. For the dynamics, periodicity plays again an important role, but this time with quite opposite effects. The manifold driven at high velocities offers no surprises and would be quite ordered (though with some anomalous KPZ type roughness) whereas the periodic system remains a glass, with surprising properties such as the existence of a transverse critical force.

The physics of disordered periodic systems offers thus a field rich of prospectives and challenging problems. Of course both for the statics and dynamics, the issue of topological defects is of paramount importance. In the statics it is the key to understand the transitions leading to the destruction of the Bragg glass, as well as the mysterious strong disorder glass. The question is also particularly important for two dimensional systems. For the dynamics the very question of the presence of topological defects in the moving glass phase is at stake. Solving this issue starting from large velocity is already a formidable task, but could help us to understand what happens close to the threshold. Indeed here again only simple cases, inspired from the manifold or CDW with scalar displacements and no transverse periodicity, have been considered previously. As in the statics it is possible that the physics is modified in a quite surprising way, and certainly all the issues about critical behavior close to threshold, dynamics reordering, elastic to plastic motion transitions, will have to be reconsidered. These issues are of major theoretical concern but also of large practical importance.

Another issue which is raised is how to handle the whole complexity of all the new terms which are generated in the driven dynamics of a lattice (in order to go beyond the simpler models which have been analyzed here). Among such new terms are the random force term, additional linear and non linear terms which are allowed by symmetry, and their interplay with the quenched disorder terms which remains due to transverse periodicity. This will require to go further than the whole topics of the generation of additional non-linearities, which in manifold physics has been successfully handled through analysis of KPZ type equations and hopefully may lead to new physics.

Finally this field opened at least two Pandora's boxes at the face of the theorist. $d = 2$ has prompted for the very question of the validity of a naive renormalization group in a glassy system, when many metastable states exist. This question and idea that RG should in some way be complemented by the proper RSB procedure has been investigated subsequently in several other disordered systems. Although we still do not know how to mix these two ingredients efficiently, simple models such as the CO could be good laboratories to try, with the advantage of being directly experimentally relevant. Last but not least, the moving glass was the first hint that non potential dynamics can indeed exhibit glassy properties, a question one can ask in a more general context. The very existence of such *dissipative glasses* may seem at first sight unnatural because after all there is constant dissipation going on in the system which naturally tends to generate or increase the effective temperature and kill glassy properties. There too the situation may be more subtle and leave much room for unexpected behavior. The study of the properties of such dissipative glasses and their comparison with normal ones, will doubtless prove to be a field worth tapping.

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1. M. Kardar cond-mat/9704172 and cond-mat/9507019.
2. G. Blatter *et al.*, Rev. Mod. Phys. **66**, 1125 (1994).
3. H. Brandt, Rep. Prog. Phys. **58**, 1465 (1995).
4. M. Tinkham, *Introduction to Superconductivity* (Mc Graw Hill, New York, 1975).
5. A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).

6. P. G. de Gennes and J. Matricon, Rev. Mod. Phys. **36**, 45 (1964).
7. G. Eilenberger, Phys. Rev. **164**, 928 (1967).
8. D. Nelson, Phys. Rev. Lett. **69**, 1973 (1988).
9. A. Houghton, R. Pelcovits, and A. Sudbo, Phys. Rev. B **40**, 6763 (1989).
10. D. R. Nelson and S. Seung, Phys. Rev. B **39**, 9153 (1989).
11. D. R. Nelson and P. Le Doussal, Phys. Rev. B **42**, 10113 (1990).
12. R. Hetzel, A. Sudbo, and D. Huse, Phys. Rev. Lett. **69**, 518 (1992).
13. R. Sasik and D. Stroud, Phys. Rev. Lett. **75**, 2582 (1995).
14. E. Brezin, D. R. Nelson, and A. Thiaville, Phys. Rev. B **31**, 7124 (1985).
15. M. A. Moore and T. J. Newman, Phys. Rev. Lett. **75**, 533 (1995).
16. L. Civale and al., Phys. Rev. Lett. **67**, 648 (1991).
17. C. J. van der Beek and al., Phys. Rev. B **51**, 15492 (1995).
18. T. Hwa, P. L. Doussal, D. R. Nelson, and V. M. Vinokur, Phys. Rev. Lett. **71**, 3545 (1993).
19. V. Hardy and al., Physica C **257**, 16 (1996).
20. D. R. Nelson and V. M. Vinokur, Phys. Rev. B **48**, 13060 (1993).
21. T. Giamarchi and P. Le Doussal, Phys. Rev. B **53**, 15206 (1996).
22. A. I. Larkin and Y. N. Ovchinnikov, J. Low Temp. Phys **34**, 409 (1979).
23. A. I. Larkin, Sov. Phys. JETP **31**, 784 (1970).
24. A. P. Malozemoff and al., Phys. Rev. B **38**, 7203 (1988).
25. R. Koch and al., Phys. Rev. Lett. **63**, 1511 (1989).
26. M. Charalambous, J. Chaussy, and P. Lejay, Phys. Rev. B **45**, 5091 (1992).
27. H. Safar *et al.*, Phys. Rev. Lett. **70**, 3800 (1993).
28. E. Zeldov and Al., Nature **375**, 373 (1995).
29. A. Schilling, R. A. Fisher, and G. W. Crabtree, Nature **382**, 791 (1996).
30. M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).
31. M. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).
32. D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1990).
33. J. L. Cardy and S. Ostlund, Phys. Rev. B **25**, 6899 (1982).
34. E. M. Chudnovsky, Phys. Rev. B **40**, 11355 (1989).
35. J.P. Bouchaud, M. Mézard and J.S. Yedidia, Phys. Rev. Lett. **67**, 3840 (1991); Phys. Rev. B **46**, 14686 (1992).
36. J. Villain and J. F. Fernandez, Z Phys. B **54**, 139 (1984).
37. D. G. Grier *et al.*, Phys. Rev. Lett. **66**, 2270 (1991).
38. H. S. Bokil and A. P. Young, Phys. Rev. Lett. **74**, 3021 (1995).
39. T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990).
40. G. Grüner, Rev. Mod. Phys. **60**, 1129 (1988).
41. E. Y. Andrei and al., Phys. Rev. Lett. **60**, 2765 (1988).
42. R. Seshadri and R. M. Westervelt, Phys. Rev. B **46**, 5142 (1992).
43. R. Seshadri and R. M. Westervelt, Phys. Rev. B **46**, 5150 (1992).
44. V. M. Vinokur and A. E. Koshelev, Sov. Phys. JETP **70**, 547 (1990).
45. L. Balents and S. H. Simon, Phys. Rev. B **51**, 6515 (1995).

46. J. Toner and D. DiVincenzo, Phys. Rev. B **41**, 632 (1990).
47. V. L. Pokrovsky and A. L. Talapov, Phys. Rev. Lett. **42**, 65 (1979).
48. T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. **72**, 1530 (1994).
49. T. Giamarchi and P. Le Doussal, Phys. Rev. B **52**, 1242 (1995).
50. H. Fukuyama and P. A. Lee, Phys. Rev. B **17**, 535 (1978).
51. U. Yaron and et al., Phys. Rev. Lett. **73**, 2748 (1994).
52. T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. **75**, 3372 (1995).
53. P. Le Doussal and V. M. Vinokur, Physica C **254**, 63 (1995).
54. S. E. Korshunov, Phys. Rev. B **48**, 3969 (1993).
55. M. Mezard and G. Parisi, J. de Phys. I **1**, 809 (1991).
56. D. S. Fisher, Phys. Rev. Lett. **56**, 1964 (1986).
57. L. Balents and D. S. Fisher, Phys. Rev. B **48**, 5959 (1993).
58. O. Narayan and D. S. Fisher, Phys. Rev. B **46**, 11520 (1992).
59. J. P. Bouchaud, M. Mézard, and J. S. Yedidia, Phys. Rev. B **46**, 14686 (1992).
60. P. Mitra and P. Le Doussal, Phys. Rev. B **44**, 12035 (1991).
61. M. J. P. Gingras and D. A. Huse, Phys. Rev. B **53**, 15193 (1996).
62. S. Ryu, A. Kapitulnik, and S. Doniach, Phys. Rev. Lett. **77**, 2300 (1996), for an early discussion of the possible phase diagram see also A. Kapitulnik et al. SPIE proceedings 2157 12 (1994).
63. J. Kierfeld, T. Nattermann, and T. Hwa, Phys. Rev. B **55**, 626 (1997).
64. D. Carpentier, P. Le Doussal, and T. Giamarchi, Europhys. Lett. **35**, 379 (1996).
65. D. S. Fisher, Phys. Rev. Lett. **78**, 1964 (1997).
66. T. Giamarchi and P. Le Doussal, Phys. Rev. B **55**, 6577 (1997).
67. R. Cubbit and al., Nature **365**, 407 (1993).
68. B. Khaykovich and et al., Phys. Rev. Lett. **76**, 2555 (1996).
69. N. Chikumoto, M. Konczykowski, N. Motohira, and A. Malozemoff, Phys. Rev. Lett. **69**, 1260 (1992).
70. Y. Y. Goldschmidt and B. Schaub, Nucl. Phys. B **251**, 77 (1985).
71. Y. Y. Goldschmidt and A. Houghton, Nucl. Phys. B **210**, 155 (1982).
72. T. Hwa and D. S. Fisher, Phys. Rev. Lett. **72**, 2466 (1994).
73. T.Nattermann, I.Lyuksyutov, and M.Schwartz, Europhys. Lett. **16**, 295 (1991).
74. D. Carpentier and P. Le Doussal, Phys. Rev. B **55**, 12128 (1997).
75. C. Carraro and D. R. Nelson, 1997, to be published in Phys. Rev. E **56** (1997).
76. Y. C. Tsai and Y. Shapir, Phys. Rev. Lett. **69**, 1773 (1992).
77. P. Le Doussal and T. Giamarchi, Phys. Rev. Lett. **74**, 606 (1995).
78. J. Kierfeld, J. de Phys. I **5**, 379 (1995).
79. G. G. Batrouni and T. Hwa, Phys. Rev. Lett. **72**, 4133 (1994).
80. D. Cule and Y. Shapir; Phys. Rev. Lett. **74**, 114 (1995) and Phys. Rev. B **51**, 3305 (1995). H. Rieger, Phys. Rev. Lett. **74**, 4964 (1995).
81. E. Marinari, R. Monasson, and J. J. Ruiz-Lorenzo, J. Phys. A **28**, 3975 (1995).
82. D. J. Lancaster and J. J. Ruiz-Lorenzo, J. Phys. A **28**, L577 (1995).
83. H. Rieger and U. Blasum, Phys. Rev. B **55**, R7394 (1997).
84. C. Zeng, A. Middleton, and Y. Shapir, Phys. Rev. Lett. **77**, 3204 (1996).

85. The two dimensional q-states Potts model is the only presently known case where a new perturbative RSB fixed point is known through a standard $q-2$ expansion see Dotsenko, Dotsenko, Pujol, Europhys. Lett. **32** 425 (1995). There it is found that the difference in the critical exponents from the RS fixed point is small (but non zero).
86. M. Rubinstein, B. Schraiman, and D. Nelson, Phys. Rev. B **27**, 1800 (1983).
87. D. R. Nelson, Phys. Rev. B **27**, 2902 (1983).
88. see the end of Section VI. B in [49].
89. T.Nattermann, S.Scheidl, S.E. Korshunov, and M.S. Li, J.Phys. I (France) **5**, 565 (1995), S.E. Korshunov and T.Nattermann, Phys. Rev. B **53**, 2746 (1996), L.H. Tang, preprint cond-mat/9602162, S.Scheidl, preprint cond-mat/961131.
90. C. A. Murray, W. O. Sprenger, and R. Wenk, Phys. Rev. B **42**, 688 (1990).
91. J. Krim and G. Palasantzas, Int. J. Mod. Phys. B **9**, 599 (1995).
92. M. Kardar, G. Parisi, and Y. Zhang, Phys. Rev. Lett. **56**, 889 (1996).
93. S. N. Coppersmith and A. J. Millis, Phys. Rev. B **44**, 7799 (1991).
94. A. C. Shi and A. J. Berlinsky, Phys. Rev. Lett. **67**, 1926 (1991).
95. M. J. Higgins and S. Bhattacharya, Physica C **257**, 232 (1996).
96. S. Bhattacharya and M. J. Higgins, Phys. Rev. B **49**, 10005 (1994).
97. A. C. Marley, M. J. Higgins, and S. Bhattacharya, Phys. Rev. Lett. **74**, 3029 (1995).
98. S. Bhattacharya and M. J. Higgins, Phys. Rev. B **52**, 64 (1995).
99. M. C. Hellerqvist and Al., Phys. Rev. Lett. **76**, 4022 (1996).
100. M. C. Hellerqvist and A. Kapitulnik, 1997, stanford Preprint.
101. D. D. N. Gronbech-Jensen, A. R. Bishop, Phys. Rev. Lett. **76**, 2985 (1996).
102. H. J. Jensen, A. Brass, and A. J. Berlinsky, Phys. Rev. Lett. **60**, 1676 (1988).
103. H. J. Jensen, A. Brass, Y. Brecht, and A. J. Berlinsky, Phys. Rev. B **38**, 9235 (1988).
104. J. Watson and D. S. Fisher preprint cond-mat/9610095.
105. See e.g. the review by L. F. Cugliandolo, J. Kurchan, J. P. Bouchaud and M. Mezard in this book.
106. D. Fisher, Phys. Rev. B **31**, 1396 (1985).
107. O. Narayan and D. Fisher, Phys. Rev. B **48**, 7030 (1993).
108. T. Nattermann, S. Stepanow, L. H. Tang, and H. Leschom, J. Phys. (Paris) **2**, 1483 (1992).
109. R. Thorel and Al., J. Phys. (Paris) **34**, 447 (1973).
110. A. I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP **38**, 854 (1974).
111. A. Schmidt and W. Hauger, J. Low Temp. Phys **11**, 667 (1973).
112. A. E. Koshelev and V. M. Vinokur, Phys. Rev. Lett. **73**, 3580 (1994).
113. T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. **76**, 3408 (1996).
114. T. Giamarchi and P. Le Doussal, 1997, to be published.
115. P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1973).
116. L. F. Cugliandolo, J. Kurchan, and P. Le Doussal, Phys. Rev. Lett. **73**, 2390 (1996).
117. L. Cugliandolo and P. Le Doussal, Phys. Rev. E **53**, 1525 (1996).
118. L. F. Cugliandolo and J. Kurchan; Phys. Rev. Lett. **71**, 173 (1993); Phil. Mag. **B71** (1995).
119. O. Narayan and D. S. Fisher, Phys. Rev. Lett. **68**, 3615 (1992).

120. T. Giamarchi and P. Le Doussal, proceedings of the M2S97 Conference (Beijing, 1997), to be published in Physica C.
121. L. Chen et al. to be published in Phys. Rev. B.
122. L. Balents and M. P. A. Fisher, Phys. Rev. Lett. **75**, 4270 (1995).
123. K. Moon and al., Phys. Rev. Lett. **77**, 2378 (1997).
124. M. Marchevsky and al., Phys. Rev. Lett. **78**, 531 (1997).
125. L. Balents, C. Marchetti and L. Radzihovsky, Phys. Rev. Lett. **78** 751 (1997); T. Giamarchi and P. Le Doussal Phys. Rev. Lett. **78** 752 (1997).
126. S. Ryu and al., Phys. Rev. Lett. **77**, 5114 (1997).
127. A. E. Koshelev, private communication.
128. K. Holmlund, 1997, *flux line channels and reduced Hall effect in Impure 2D superconductors*, cond-mat/9603061.
129. L. F. Cugliandolo, J. Kurchan, P. Le Doussal, and L. Peliti, Phys. Rev. Lett. **78**, 350 (1997).
130. P. Le Doussal, K. Wiese, in preparation.
131. J.P. Bouchaud, A. Comtet, A. Georges, P. Le Doussal, J. Physique (Paris) **48** 1445 (1987) and **49** 369 (1988).
132. P. Le Doussal, L. F. Cugliandolo, L. Peliti preprint cond-mat/9612079 submitted to Europhys. Lett.
133. J. T. Chalker and Z. Jane Wang preprint cond-mat/9704198